Chapter 7: Logarithmic Functions

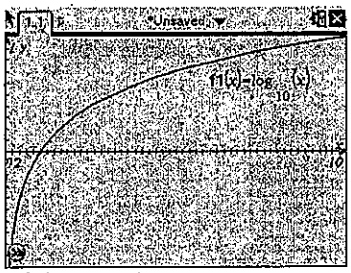
Lesson 7.1: Characteristics of Logarithmic Functions with Base 10 and Base *e*, page 420

1. *x*-intercept: 1
Number of *y*-intercepts: 0
End Behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$

2. a) No. e.g., no *x*-intercept and one *y*-intercept whereas logarithmic functions have no *y*-intercepts and one *x*-intercept. It is an exponential function.
b) No. e.g., two *x*-intercepts and one *y*-intercept whereas logarithmic functions have no *y*-intercepts and one *x*-intercept. It is a quadratic function.
c) Yes. e.g., one *x*-intercept and no *y*-intercept.
d) No. e.g., one *x*-intercept and one *y*-intercept whereas logarithmic functions have *x*-intercepts of 1.
e) Yes. e.g., one *x*-intercept and no *y*-intercept.
f) No. e.g., no *x*-intercept and one *y*-intercept whereas logarithmic functions have no *y*-intercepts and one *x*-intercept.

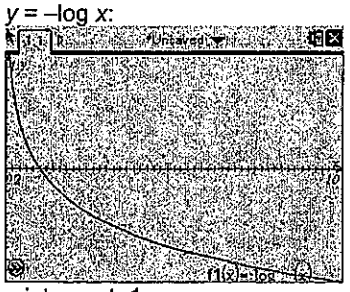
3. c: *x*-intercept: 1
Number of *y*-intercepts: 0
End Behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
a: $a > 0$ e.g., since the graph is increasing
e: *x*-intercept: 1
Number of *y*-intercepts: 0
End Behaviour: QI to QIV
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
a: $a < 0$ e.g., since the graph is decreasing

4. $y = \log x$:



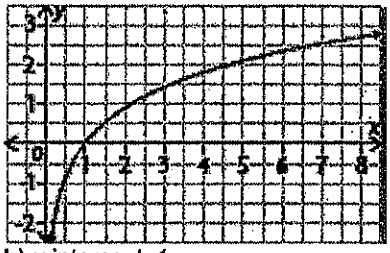
x-intercept: 1
Number of *y*-intercepts: 0
End Behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
Increasing or decreasing: increasing

$y = -\log x$:

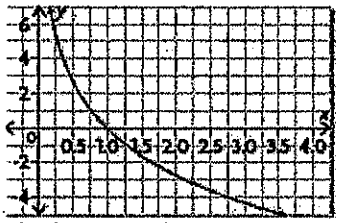


x-intercept: 1
Number of *y*-intercepts: 0
End Behaviour: QI to QIV
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
Increasing or decreasing: decreasing

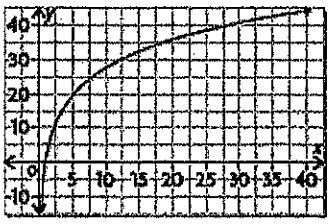
5. a) *x*-intercept: 1
Number of *y*-intercepts: 0
End Behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
Increasing or decreasing: increasing



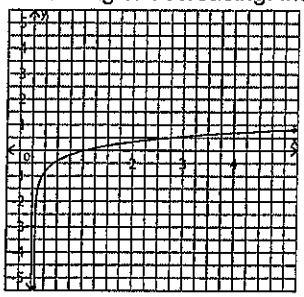
b) *x*-intercept: 1
Number of *y*-intercepts: 0
End Behaviour: QI to QIV
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
Increasing or decreasing: decreasing



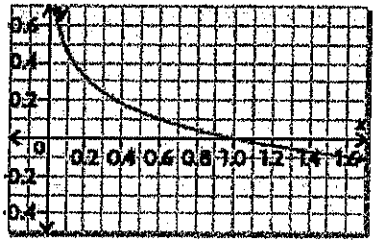
c) *x*-intercept: 1
Number of *y*-intercepts: 0
End Behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
Increasing or decreasing: increasing



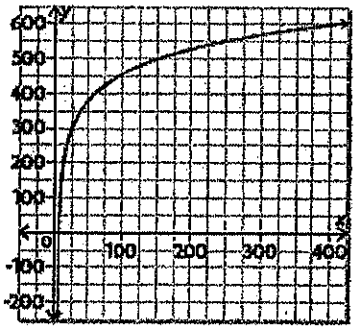
d) x-intercept: 1
Number of y-intercepts: 0
End Behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
Increasing or decreasing: increasing



e) x-intercept: 1
Number of y-intercepts: 0
End Behaviour: QI to QIV
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
Increasing or decreasing: decreasing

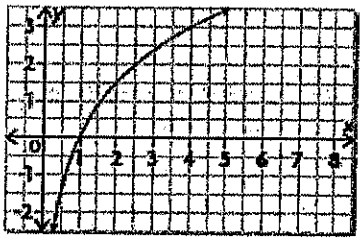


f) x-intercept: 1
Number of y-intercepts: 0
End Behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
Increasing or decreasing: increasing

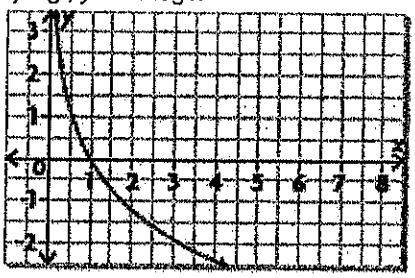


6. e.g., one x-intercept of 1, no y-intercepts, domain: $\{x \mid x > 0, x \in R\}$.

7. a) e.g., $y = 5 \log x$



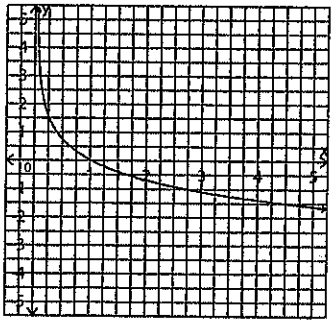
b) e.g., $y = -4 \log x$



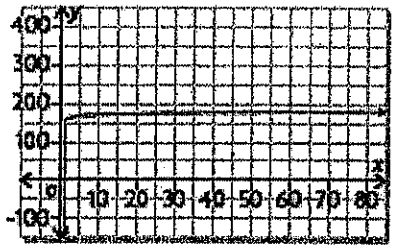
8. i) b, e.g., x-intercept is 1, no y-intercept, graph extends from QIV to QI. Thus, the function is logarithmic so b and c are the only options. The function is increasing so the correct graph is b.
ii) c, e.g., x-intercept is 1, no y-intercept, graph extends from QI to QIV. Thus, the function is logarithmic so b and c are the only options. The function is decreasing so the correct graph is c.
iii) d, e.g., no x-intercept, y-intercept is 1, graph extends from QII to QI. Thus, the function is exponential so a and d are the only options. The function is increasing so the correct graph is d.
iv) a, e.g., no x-intercept, y-intercept is 2, graph extends from QII to QI. Thus, the function is exponential a and d are the only options. The function is decreasing so the correct graph is a.

9. Yes, e.g., An exponential function has no x-intercepts, and a logarithmic function has one x-intercept.

10. As hydrogen ion concentration increases, pH decreases.



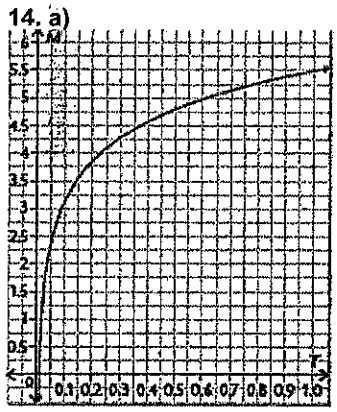
11. As the energy of the sound increases, the number of decibels increases.



12. a) P-intercept: 1, t-intercept: none, domain: $\{P \mid 0 < P \leq 1, P \in \mathrm{R}\}$, range: $\{t \mid t \geq 0, t \in \mathrm{R}\}$, function: decreasing, $P = 1$: $t = 0$.
b) $t = -100.422 \log P$
$t = -100.422 \log(0.05)$
$t = 130.652...$
It would take about 131 years for only 5% of the original amount to remain.
c) $t = -100.422 \log P$
$t = -100.422 \log(0.5)$
$t = 30.230...$
The half-life of cesium-137 is about 30 years.

13. a) When $a < 0$, each function will be decreasing.
b) The domain of each function is restricted as for each function x must be greater than 0.
c) The range is unrestricted as all values of y are possible.

14. a)



The T-intercept is $10^{-5.5}$ and there is no M-intercept. In context, the domain of this graph is $\{T \mid T \geq 10^{-5.5}, T \in \mathrm{R}\}$, and the range is $\{M \mid M \geq 0, M \in \mathrm{R}\}$. The graph is increasing, and when $T = 1$, $M = 5.5$.
b) $M = \log T + 5.5$
$M = \log(50) + 5.5$
$M = 7.198...$
An earthquake that is 50 times more intense than the Saskatchewan earthquake would have a magnitude of about 7.2.
c) This earthquake was about 3000 times as intense than the Saskatchewan earthquake.

15. a) The A-intercept is 3000 and there is no t-intercept. In context, the domain of this graph is $\{A \mid A \geq 3000, A \in \mathrm{R}\}$, and the range is $\{t \mid t \geq 0, t \in \mathrm{R}\}$. This function is increasing.
b) $t = 58.7 \log A - 204.1$
$t = 58.7 \log(10\,000) - 204.1$
$t = 30.7$
It will take about 31 years for Christie's investment to reach \$10 000.
c) $t = 58.7 \log A - 204.1$
$t = 58.7 \log(6000) - 204.1$
$t = 17.677...$
It will take about 18 years for Christie's investment to double.

Lesson 7.2: Evaluating Logarithmic Expressions, page 436

1. a) $y = \log x$
$x = 10^y$
b) $y = \ln x$
$x = e^y$
c) $y = \log_5 x$
$x = 5^y$
d) $y = \log_8 x$
$x = 8^y$

2. a) $x = e^y$
$y = \ln x$
b) $x = 10^y$
$y = \log x$
c) $x = 4^y$
$y = \log_4 x$
d) $x = 9^y$
$y = \log_9 x$

3. a) $\log 20 = 1.30$
b) $\log 45 = 1.65$
c) $\ln 40 = 3.69$
d) $\ln 98 = 4.58$

4. a) Let $y = \log_6 36$
$y = \log_6 36$
$6^y = 36$
$6^y = 6^2$
$y = 2$
$\log_6 36 = 2$
b) Let $y = \log_4 64$
$y = \log_4 64$
$4^y = 64$
$4^y = 4^3$
$y = 3$
$\log_4 64 = 3$

c) Let $y = \log_{10} 1000$
$y = \log_{10} 1000$
$10^y = 1000$
$10^y = 10^3$
$y = 3$
$\log_{10} 1000 = 3$

d) Let $y = \log_2 16$
$y = \log_2 16$
$2^y = 16$
$2^y = 2^4$
$y = 4$
$\log_2 16 = 4$

5. a) I estimate that $2 < y < 3$ since $10^2 = 100$ is less than 250 and $10^3 = 1000$ is greater than 250.
$250 = 10^y$
$y = \log 250$
$y = 2.397...$
It is approximately 2.4.
b) I estimate that $2 < y < 3$ since $e^2 < 9 < e^3$.
$9 = e^y$
$y = \ln 9$
$y = 2.197...$
It is approximately 2.2.

6. a) $\frac{1}{9} = 3^y$

$y = \log_3 \frac{1}{9}$

b) $1\,000\,000 = 2^y$
$y = \log_2 1\,000\,000$
c) $5 = 7^y$
$y = \log_7 5$
d) $x = a^y$
$y = \log_a x$

7. a) $x = \log_{\frac{1}{2}} 100$

$\left(\frac{1}{2}\right)^x = 100$

b) $x = \log_{20} 40$
$20^x = 40$
c) $x = \ln 0.25$
$e^x = 0.25$
d) $x = \ln 1000$
$e^x = 1000$

8. a) Let $y = \log_3 81$
$y = \log_3 81$
$3^y = 81$
$3^y = 3^4$
$y = 4$
$\log_3 81 = 4$
b) Let $y = \log_4 16$
$y = \log_4 16$
$4^y = 16$
$4^y = 4^2$
$y = 2$
$\log_4 16 = 2$
c) Let $y = \log_8 64$
$y = \log_8 64$
$8^y = 64$
$8^y = 8^2$
$y = 2$
$\log_8 64 = 2$

d) Let $y = \log_5 1$
$y = \log_5 1$
$5^y = 1$
$5^y = 5^0$
$y = 0$
$\log_5 1 = 0$

9. a) Let $y = \log_3 \left(\frac{1}{9}\right)$

$y = \log_3 \left(\frac{1}{9}\right)$

$3^y = \frac{1}{9}$

$3^y = 3^{-2}$

$y = -2$

$\log_3 \left(\frac{1}{9}\right) = -2$

b) Let $y = \log_2 \left(\frac{1}{8}\right)$

$y = \log_2 \left(\frac{1}{8}\right)$

$2^y = \frac{1}{8}$

$2^y = 2^{-3}$

$y = -3$

$\log_2 \left(\frac{1}{8}\right) = -3$

c) Let $y = \log_4 \left(\frac{1}{64}\right)$

$y = \log_4 \left(\frac{1}{64}\right)$

$4^y = \frac{1}{64}$

$4^y = 4^{-3}$

$y = -3$

$\log_4 \left(\frac{1}{64}\right) = -3$

d) Let $y = \log_5 \left(\frac{1}{125}\right)$

$y = \log_5 \left(\frac{1}{125}\right)$

$5^y = \frac{1}{125}$

$5^y = 5^{-3}$

$y = -3$

$\log_5 \left(\frac{1}{125}\right) = -3$

10. a) Let $y = \log_{\frac{1}{3}} 9$

$$y = \log_{\frac{1}{3}} 9$$

$$\left(\frac{1}{3}\right)^y = 9$$

$$\left(\frac{1}{3}\right)^y = \left(\frac{1}{3}\right)^{-2}$$

$$y = -2$$

$$\log_{\frac{1}{3}} 9 = -2$$

b) Let $y = \log_{\frac{1}{2}} 8$

$$y = \log_{\frac{1}{2}} 8$$

$$\left(\frac{1}{2}\right)^y = 8$$

$$\left(\frac{1}{2}\right)^y = \left(\frac{1}{2}\right)^{-3}$$

$$y = -3$$

$$\log_{\frac{1}{2}} 8 = -3$$

c) Let $y = \log_{\frac{1}{2}} \left(\frac{1}{4}\right)$

$$y = \log_{\frac{1}{2}} \left(\frac{1}{4}\right)$$

$$\left(\frac{1}{2}\right)^y = \frac{1}{4}$$

$$\left(\frac{1}{2}\right)^y = \left(\frac{1}{2}\right)^2$$

$$y = 2$$

$$\log_{\frac{1}{2}} \left(\frac{1}{4}\right) = 2$$

d) Let $y = \log_{\frac{1}{7}} \left(\frac{1}{49}\right)$

$$y = \log_{\frac{1}{7}} \left(\frac{1}{49}\right)$$

$$\left(\frac{1}{7}\right)^y = \frac{1}{49}$$

$$\left(\frac{1}{7}\right)^y = \left(\frac{1}{7}\right)^2$$

$$y = 2$$

$$\log_{\frac{1}{7}} \left(\frac{1}{49}\right) = 2$$

11. a) $\log_3 81 + \log_3 3$

First term: $\log_3 81$

$3^4 = 81$, so the first term in this expression is equal to 4.

Second term: $\log_3 3$

$3^1 = 3$, so the second term in this expression is equal to 1.

$\log_3 81 + \log_3 3 = 4 + 1$

$= 5$

b) $\log_2 64 - \log_2 4$

First term: $\log_2 64$

$2^6 = 64$, so the first term in this expression is equal to 6.

Second term: $\log_2 4$

$2^2 = 4$, so the second term in this expression is equal to 2.

$\log_2 64 - \log_2 4 = 6 - 2$

$= 4$

c) $\left(\log_4 1\right)\left(\log_5 \left(\frac{1}{5}\right)\right)$

First term: $\log_4 1$

$4^0 = 1$, so the first term in this expression is equal to 0.

Second term: $\log_5 \left(\frac{1}{5}\right)$

$5^{-1} = \frac{1}{5}$, so the second term in this expression is equal to -1.

$\left(\log_4 1\right)\left(\log_5 \left(\frac{1}{5}\right)\right) = (0)(-1)$

$= 0$

d) $\log_3 27 \div \log_3 \left(\frac{1}{9}\right)$

First term: $\log_3 27$

$3^3 = 27$, so the first term in this expression is equal to 3.

Second term: $\log_3 \left(\frac{1}{9}\right)$

$5^{-2} = \frac{1}{9}$, so the second term in this expression is equal to -2.

$\log_3 27 \div \log_3 \left(\frac{1}{9}\right) = 3 \div (-2)$

$\log_3 27 \div \log_3 \left(\frac{1}{9}\right) = -\frac{3}{2}$

12. A: $\log_2 32 - \log_2 8$

First term: $\log_2 32$

$2^5 = 32$, so the first term in expression A is equal to 5.

Second term: $\log_2 8$

$2^3 = 8$, so the second term in expression A is equal to 3.

$\log_2 32 - \log_2 8 = 5 - 3$

$= 2$

B: $\log 85 + \log 5 = 2.628...$

Since $2.628... > 2$, B is greater than A.

13. A: $\log_{\left(\frac{1}{2}\right)}\left(\frac{1}{16}\right)+\log_2\left(\frac{1}{8}\right)$

First term: $\log_{\left(\frac{1}{2}\right)}\left(\frac{1}{16}\right)$

$\left(\frac{1}{2}\right)^4 = \frac{1}{16}$, so the first term in expression A is equal to 4.

Second term: $\log_2\left(\frac{1}{8}\right)$

$2^{-3} = \frac{1}{8}$, so the second term in expression A is equal to –3.

$$\log_{\left(\frac{1}{2}\right)}\left(\frac{1}{16}\right)+\log_2\left(\frac{1}{8}\right)=4+(-3)$$

$$\log_{\left(\frac{1}{2}\right)}\left(\frac{1}{16}\right)+\log_2\left(\frac{1}{8}\right)=1$$

B: $\log_3 27 + \ln 2$
First term: $\log_3 27$
$3^3 = 27$, so the first term in expression B is equal to 3.
$\log_3 27 + \ln 2 = 3 + 0.693\ldots$
$= 3.693\ldots$
C: $\log 100 + \log_8 8$
First term: $\log 100$
$10^2 = 100$, so the first term in expression C is 2.
Second term: $\log_8 8$
$8^1 = 8$, so the second term in expression C is 1.
$\log 100 + \log_8 8 = 2 + 1$
$= 3$
The order of the expressions from least to greatest is A, C, B.

14. a) $pH = -\log[H^+]$
$pH = -\log[1.00]$
$pH = 0$
The pH of this solution is 0.
b) $pH = -\log[H^+]$
$pH = -\log[1.58 \times 10^{-2}]$
$pH = 1.801\ldots$
The pH of this solution is 1.8.
c) $pH = -\log[H^+]$
$pH = -\log[6.05 \times 10^{-4}]$
$pH = 3.218\ldots$
The pH of this solution is 3.2.
d) $pH = -\log[H^+]$
$pH = -\log[2.13 \times 10^{-10}]$
$pH = 9.671\ldots$
The pH of this solution is 9.7.
e) $pH = -\log[H^+]$
$pH = -\log[4.87 \times 10^{-12}]$
$pH = 11.312\ldots$
The pH of this solution is 11.3.
f) $pH = -\log[H^+]$
$pH = -\log[8.43 \times 10^{-8}]$
$pH = 7.074\ldots$
The pH of this solution is 7.1.

15. a) $pH = -\log[H^+]$
$3.1 = -\log[H^+]$
$-3.1 = \log[H^+]$
$[H^+] = 10^{-3.1}$
$[H^+] = 7.943\ldots \times 10^{-4}$
The concentration of hydrogen ions in apple juice is $7.943\ldots \times 10^{-4}$ mol/L.
b) $pH = -\log[H^+]$
$4.2 = -\log[H^+]$
$-4.2 = \log[H^+]$
$[H^+] = 10^{-4.2}$
$[H^+] = 6.309\ldots \times 10^{-5}$
The concentration of hydrogen ions in tomato juice is $6.309\ldots \times 10^{-5}$ mol/L.
c) $pH = -\log[H^+]$
$6.5 = -\log[H^+]$
$-6.5 = \log[H^+]$
$[H^+] = 10^{-6.5}$
$[H^+] = 3.162\ldots \times 10^{-7}$
The concentration of hydrogen ions in milk is $3.162\ldots \times 10^{-7}$ mol/L.
d) $pH = -\log[H^+]$
$7.4 = -\log[H^+]$
$-7.4 = \log[H^+]$
$[H^+] = 10^{-7.4}$
$[H^+] = 3.981\ldots \times 10^{-8}$
The concentration of hydrogen ions in a swimming pool water is $3.981\ldots \times 10^{-8}$ mol/L.
e) $pH = -\log[H^+]$
$7.8 = -\log[H^+]$
$-7.8 = \log[H^+]$
$[H^+] = 10^{-7.8}$
$[H^+] = 1.584\ldots \times 10^{-8}$
The concentration of hydrogen ions in a raw egg is $1.584\ldots \times 10^{-8}$ mol/L.

16. a) Car battery acid has a pH of 0, and distilled water has a pH of 7.
Car battery acid:
$pH = -\log[H^+]_C$
$0 = -\log[H^+]_C$
$0 = \log[H^+]_C$
$[H^+]_C = 10^0$
$[H^+]_C = 1$
Distilled water:
$pH = -\log[H^+]_D$
$7 = -\log[H^+]_D$
$-7 = \log[H^+]_D$
$[H^+]_D = 10^{-7}$

$$\frac{[H^-]_C}{[H^-]_D} = \frac{1}{10^{-7}}$$

$$\frac{[H^-]_C}{[H^-]_D} = 10^7$$

$$\frac{[H^-]_C}{[H^-]_D} = 10000000$$

The acid used in car batteries is 10 million times more acidic than distilled water.

b) Distilled water has a pH of 7, and ammonia solution has a pH of 11.
Distilled water:
$pH = -\log[H^+]_D$
$7 = -\log[H^+]_D$
$-7 = \log[H^+]_D$
$[H^+]_D = 10^{-7}$
Ammonia solution:
$pH = -\log[H^+]_A$
$11 = -\log[H^+]_A$
$-11 = \log[H^+]_A$
$[H^+]_A = 10^{-11}$

$$\frac{[H^-]_D}{[H^-]_A} = \frac{10^{-7}}{10^{-11}}$$

$$\frac{[H^-]_D}{[H^-]_A} = 10^{(-7-(-11))}$$

$$\frac{[H^-]_D}{[H^-]_A} = 10^4$$

$$\frac{[H^-]_D}{[H^-]_A} = 10000$$

Distilled water is 10 000 times more acidic than ammonia solution.

17. Tap water has a pH of 6.3, and distilled water has a pH of 007.
Tap water:
$pH = -\log[H^+]_T$
$6.3 = -\log[H^+]_T$
$-6.3 = \log[H^+]_T$
$[H^+]_T = 10^{-6.3}$
Distilled water:
$pH = -\log[H^+]_D$
$7 = -\log[H^+]_D$
$-7 = \log[H^+]_D$
$[H^+]_D = 10^{-7}$

$$\frac{[H^-]_T}{[H^-]_D} = \frac{10^{-6.3}}{10^{-7}}$$

$$\frac{[H^-]_T}{[H^-]_D} = 10^{(-6.3-(-7))}$$

$$\frac{[H^-]_T}{[H^-]_D} = 10^{(-6.3+7)}$$

$$\frac{[H^-]_T}{[H^-]_D} = 10^{0.7}$$

$$\frac{[H^-]_T}{[H^-]_D} = 5.011...$$

Tap water is about 5.0 times more acidic than distilled water.

18. a) $\beta = 10(\log I + 12)$
$\beta = 10(\log (1 \times 10^{-11}) + 12)$
$\beta = 10(-11 + 12)$
$\beta = 10(1)$
$\beta = 10$
The sound level of rustling leaves is 10 dB.

b) $\beta = 10(\log I + 12)$
$\beta = 10(\log (1 \times 10^{-2}) + 12)$
$\beta = 10(-2 + 12)$
$\beta = 10(10)$
$\beta = 100$
The sound level of a siren at 30 m is 100 dB.
c) $\beta = 10(\log I + 12)$
$\beta = 10(\log 1 + 12)$
$\beta = 10(0 + 12)$
$\beta = 10(12)$
$\beta = 120$
The sound level of the threshold of pain is 120 dB.
d) $\beta = 10(\log I + 12)$
$\beta = 10(\log (3 \times 10^{-6}) + 12)$
$\beta = 10(-5.522... + 12)$
$\beta = 10(6.477...)$
$\beta = 64.771...$
The sound level of an ordinary conversation at 50 cm is 65 dB.

19. The sound level of a loud indoor rock concert is 80 dB louder than a quiet radio. Therefore, it is 10^8, or 100 million times as loud.

20. The equation that can be used to represent the Richter scale is $R = \log a$, where a is the amplitude of vibrations, and R is the Richter scale magnitude. The Richter scale magnitude of the Honshu earthquake was 9, and the magnitude of the Burwash Landing earthquake was 4.2.
Honshu Earthquake:
$R = \log a_H$
$9 = \log a_H$
$a_H = 10^9$
Burwash Landing Earthquake:
$R = \log a_B$
$4.2 = \log a_B$
$a_B = 10^{4.2}$

$$\frac{a_H}{a_B} = \frac{10^9}{10^{4.2}}$$

$$\frac{a_H}{a_B} = 10^{(9-4.2)}$$

$$\frac{a_H}{a_B} = 10^{4.8}$$

$$\frac{a_H}{a_B} = 63\ 095.734...$$

The Honshu earthquake released about 63 000 times more energy than the Burwash Landing earthquake.

21. e.g., To evaluate $\log_b A$, determine the exponent needed that when applied to the base b gives the result A. If A is a power of the base b, the exponent can be determined without technology.

22. a) Let $y = \log_3 \sqrt{3}$

$$y = \log_3 \sqrt{3}$$
$$3^y = \sqrt{3}$$
$$3^y = 3^{\frac{1}{2}}$$
$$y = \frac{1}{2}$$
$$\log_3 \sqrt{3} = \frac{1}{2}$$

b) Let $y = \log_4 \sqrt[3]{4}$

$$y = \log_4 \sqrt[3]{4}$$
$$4^y = \sqrt[3]{4}$$
$$4^y = 4^{\frac{1}{3}}$$
$$y = \frac{1}{3}$$
$$\log_4 \sqrt[3]{4} = \frac{1}{3}$$

c) Let $y = \log_2 \sqrt{\frac{1}{8}}$

$$y = \log_2 \sqrt{\frac{1}{8}}$$
$$2^y = \sqrt{\frac{1}{8}}$$
$$2^y = \left(2^{-3}\right)^{\frac{1}{2}}$$
$$2^y = 2^{-\frac{3}{2}}$$
$$y = -\frac{3}{2}$$
$$\log_2 \sqrt{\frac{1}{8}} = -\frac{3}{2}$$

d) Let $y = \log_5 \sqrt[3]{\frac{1}{125}}$

$$y = \log_5 \sqrt[3]{\frac{1}{125}}$$
$$y = \log_5 \sqrt[3]{\left(\frac{1}{5}\right)^3}$$
$$y = \log_5 \left(\frac{1}{5}\right)$$
$$5^y = \frac{1}{5}$$
$$5^y = 5^{-1}$$
$$y = -1$$
$$\log_5 \sqrt[3]{\frac{1}{125}} = -1$$

23. a) $\log T = \left(\frac{3}{2}\right) \log r - 3.264$

$$\log T = \left(\frac{3}{2}\right) \log 108 - 3.264$$
$$\log T = 3.050\ldots - 3.264$$
$$\log T = -0.213\ldots$$
$$T = 10^{-0.213\ldots}$$
$$T = 0.611\ldots$$

It takes Venus 0.61 Earth years to orbit the sun.

b) $\log T = \left(\frac{3}{2}\right) \log r - 3.264$

$$\log T = \left(\frac{3}{2}\right) \log 228 - 3.264$$
$$\log T = 3.536\ldots - 3.264$$
$$\log T = 0.272\ldots$$
$$T = 10^{0.272\ldots}$$
$$T = 1.874\ldots$$

It takes Mars 1.87 Earth years to orbit the sun.

c) $\log T = \left(\frac{3}{2}\right) \log r - 3.264$

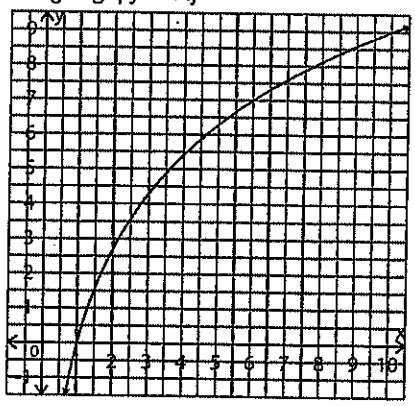
$$\log T = \left(\frac{3}{2}\right) \log 1\,430 - 3.264$$
$$\log T = 4.733\ldots - 3.264$$
$$\log T = 1.469\ldots$$
$$T = 10^{1.469\ldots}$$
$$T = 29.444\ldots$$

It takes Saturn 29.44 Earth years to orbit the sun.

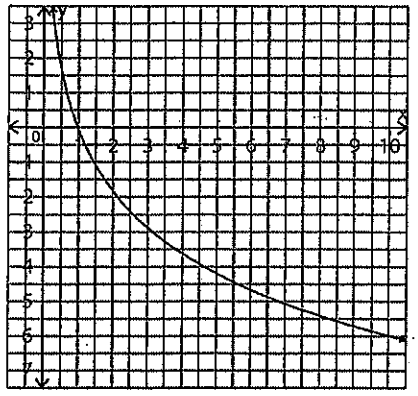
Mid-Chapter Review, page 441

1. x-intercept: 1
Number of y-intercepts: none
End Behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$

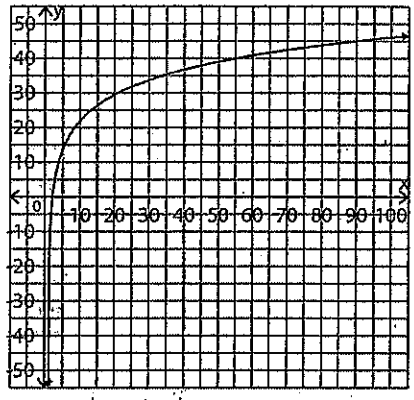
2. a) x-intercept: 1
Number of y-intercepts: none
End Behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$



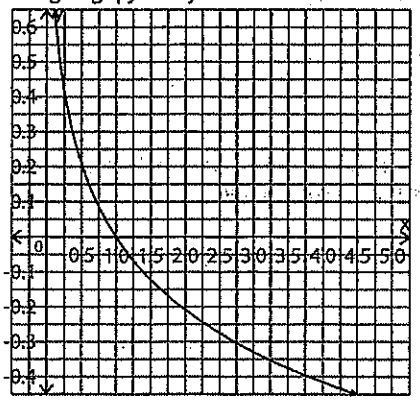
b) *x*-intercept: 1
Number of *y*-intercepts: none
End Behaviour: QI to QIV
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$



c) *x*-intercept: 1
Number of *y*-intercepts: none
End Behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$



d) *x*-intercept: 1
Number of *y*-intercepts: none
End Behaviour: QI to QIV
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$



3. i) d, e.g., *x*-intercept of 1, no *y*-intercept, QI to QIV. Thus, the function is logarithmic so a and d are the only possible choices. The function has a negative leading coefficient, which means that the function is a decreasing function and therefore the correct choice is d.
ii) b, e.g., no *x*-intercept, *y*-intercept of 1, increasing. Thus. the function is exponential so b and c are the only possible choices. Since $b > 1$, the function is increasing and therefore the correct choice is b.
iii) a, e.g., *x*-intercept of 1, no *y*-intercept, QIV to QI. Thus, the function is logarithmic so a and d are the only possible choices. The function has a positive leading coefficient which means that the function is an increasing function and therefore the correct choice is a.
iv) c, e.g., no *x*-intercept, *y*-intercept of 6, decreasing. Thus, the function is exponential so b and c are the only possible choices. Since $b < 1$, the function is decreasing and therefore the correct choice is c.

4. a) $y = \log_8 x$
$x = 8^y$
b) $y = \log_3 x$
$x = 3^y$

5. a) $x = 5^y$
$y = \log_5 x$
b) $x = 9^y$
$y = \log_9 x$

6. a) log 75 = 1.875...
It is approximately 1.88.
b) ln 22 = 3.091...
It is approximately 3.09.

7. a) Let $y = \log_4 16$
$y = \log_4 16$
$4^y = 16$
$4^y = 4^2$
$y = 2$
$\log_4 16 = 2$
b) Let $y = \log_3 243$
$y = \log_3 243$
$3^y = 243$
$3^y = 3^5$
$y = 5$
$\log_3 243 = 5$
c) let $y = \log 10$
$y = \log 10$
$10^y = 10$
$y = 1$
$\log 10 = 1$
d) Let $y = \log_5 1$
$y = \log_5 1$
$5^y = 1$
$5^y = 5^0$
$y = 0$
$\log_5 1 = 0$

8. a) Let $y = \log_3\left(\frac{1}{27}\right)$

$$y = \log_3\left(\frac{1}{27}\right)$$

$$3^y = \frac{1}{27}$$

$$3^y = (3)^{-3}$$

$$y = -3$$

$$\log_3\left(\frac{1}{27}\right) = -3$$

b) Let $y = \log_2\left(\frac{1}{4}\right)$

$$y = \log_2\left(\frac{1}{4}\right)$$

$$2^y = \frac{1}{4}$$

$$2^y = (2)^{-2}$$

$$y = -2$$

$$\log_2\left(\frac{1}{4}\right) = -2$$

c) Let $y = \log_{\left(\frac{1}{4}\right)} 64$

$$y = \log_{\left(\frac{1}{4}\right)} 64$$

$$\left(\frac{1}{4}\right)^y = 64$$

$$\left(\frac{1}{4}\right)^y = \left(\frac{1}{4}\right)^{-3}$$

$$y = -3$$

$$\log_{\left(\frac{1}{4}\right)} 64 = -3$$

d) Let $y = \log_{\left(\frac{1}{5}\right)} 25$

$$y = \log_{\left(\frac{1}{5}\right)} 25$$

$$\left(\frac{1}{5}\right)^y = 25$$

$$\left(\frac{1}{5}\right)^y = \left(\frac{1}{5}\right)^{-2}$$

$$y = -2$$

$$\log_{\left(\frac{1}{5}\right)} 25 = -2$$

9. A. $2\log_3 27 - \log_{\left(\frac{1}{3}\right)} 81$

First term: $2 \log_3 27$

$3^3 = 27$, so the first term in this expression is equal to $(2)(3) = 6$.

Second term: $\log_{\left(\frac{1}{3}\right)} 81$

$\left(\frac{1}{3}\right)^{-4} = 81$, so the second term in this expression is equal to -4.

$$2\log_3 27 - \log_{\left(\frac{1}{3}\right)} 81 = 6 - (-4)$$

$$2\log_3 27 - \log_{\left(\frac{1}{3}\right)} 81 = 10$$

B. $\log_7\left(\frac{1}{7}\right) + \log_{\left(\frac{1}{5}\right)} 125$

First term: $\log_7\left(\frac{1}{7}\right)$

$7^{-1} = \frac{1}{7}$, so the first term in this expression is equal to -1.

Second term: $\log_{\left(\frac{1}{5}\right)} 125$

$\left(\frac{1}{5}\right)^{-3} = 125$, so the second term in this expression is equal to -3.

$$\log_7\left(\frac{1}{7}\right) + \log_{\left(\frac{1}{5}\right)} 125 = -1 + (-3)$$

$$\log_7\left(\frac{1}{7}\right) + \log_{\left(\frac{1}{5}\right)} 125 = -4$$

Therefore, A. is the expression with greater value.

10. a) $p(x) = -\log x$

$p(0.005) = -\log (0.005)$

$p(0.005) = 2.301\ldots$

Therefore the pH of the solution is 2.3.

b) $p(x) = -\log x$

$7.0 = -\log x$

$-7.0 = \log x$

$x = 10^{-7.0}$

$x = 0.000\ 000\ 1$

Therefore the concentration of hydrogen ions in distilled water is 0.000 000 1 mol/L.

c) Black coffee:

$p(x_C) = -\log x_C$

$5 = -\log x_C$

$-5 = \log x_C$

$x_C = 10^{-5}$

Bleach:

$p(x_B) = -\log x_B$

$13 = -\log x_B$

$-13 = \log x_B$

$x_B = 10^{-13}$

$\frac{x_C}{x_B} = \frac{10^{-5}}{10^{-13}}$

$\frac{x_C}{x_B} = \frac{1}{10^{-8}}$

$\frac{x_C}{x_B} = 10^8$

$\frac{x_C}{x_B} = 100\,000\,000$

Therefore black coffee is 100 000 000 times more acidic than bleach.

Lesson 7.3: Laws of Logarithms, page 446

1. a) $\log_5(8 \cdot 6) = \log_5 8 + \log_5 6$

b) $\log_z(xy) = \log_z x + \log_z y$

2. a) $\log_3 7 + \log_3 5 = \log_3(7 \cdot 5)$

b) $\log_b s + \log_b r = \log_b(sr)$

3. a) $\log_2\left(\frac{36}{11}\right) = \log_2 36 - \log_2 11$

b) $\log_z\left(\frac{x}{y}\right) = \log_z x - \log_z y$

4. a) $\log_3 15 - \log_3 11 = \log_3\left(\frac{15}{11}\right)$

b) $\log_b s - \log_b r = \log_b\left(\frac{s}{r}\right)$

5. a) $\log_6 7^3 = 3\log_6 7$

b) $\log_z x^y = y\log_z x$

6. a) $\log_6 2 + \log_6 108 = \log_6(2 \cdot 108)$
$= \log_6 216$
$= 3$

b) $\log_3 810 - \log_3 20 = \log_3 \frac{810}{10}$
$= \log_3 81$
$= 4$

c) $\log 5 + \log 0.002 = \log(5 \cdot 0.002)$
$= \log \frac{1}{100}$
$= -2$

d) $\log_2 16 - \log_2 2 = \log_2 \frac{16}{2}$
$= \log_2 8$
$= 3$

7. a) $\left(\frac{1}{2}\right)\log_3 36 - \log_3 2 = \log_3 6 - \log_3 2$
$= \log_3\left(\frac{6}{2}\right)$
$= \log_3 3$
$= 1$

b) $\log 625 + 2\log 4 = \log 625 + \log 16$
$= \log(625 \cdot 16)$
$= \log 10\,000$
$= 4$

c) $\log_2 40 - \log_2 2.5 = \log_2\left(\frac{40}{2.5}\right)$
$= \log_2 16$
$= 4$

d) $\log_3 27 - \left(\frac{1}{3}\right)\log_3 27 = \log_3 27 - \log_3 3$
$= \log_3\left(\frac{27}{3}\right)$
$= \log_3 9$
$= 2$

8. a) e.g., $\log_2 32 = \log_2 8 + \log_2 4$
$\log_2 32 = \log_2 64 - \log_2 2$

b) e.g., $\log 125 = \log 25 + \log 5$
$\log 125 = \log 250 - \log 2$

9. a) e.g., $\log_2 32 = 5\log_2 2$

b) $\log 125 = 3\log 5$

10. e.g., Daniel made an error when he simplified $(2^y)^3$ to 2^{y+3}. The last three lines should be:
$2^{3y} = 5^3$ I used the exponent law for the power of a power.
$3y = \log_2 5^3$ I wrote the equation in logarithmic form.
$3\log_2 5 = \log_2 5^3$ I replaced y with the logarithm it represented.

11. a) $\log_3 2 + \log_3 4.5 = \log_3(2 \cdot 4.5)$
$= \log_3 9$
$= 2$

b) $\log_3 162 - \log_3 2 = \log_3\left(\frac{162}{2}\right)$
$= \log_3 81$
$= 4$

c) $\log_2 16^3 = 3\log_2 16$
$= 3 \cdot 4$
$= 12$

d) $\log_5 10 - \log_5 1250 = \log_5\left(\frac{10}{1250}\right)$
$= \log_5\left(\frac{1}{125}\right)$
$= -3$

12. a) $\log 40 - \log 8 = \log\left(\frac{40}{8}\right)$

$= \log 5$

$= 0.698...$

It is approximately 0.70.

b) $\log 5 + 2\log 2 = \log 5 + \log 4$

$= \log (5 \cdot 4)$

$= \log 20$

$= 1.301...$

It is approximately 1.30.

13. $3\log_8 2 - \left(\frac{1}{2}\right)\log_8 64 = \log_8 8 - \log_8 8$

$= 0$

14. e.g., Wayne made an error when he simplified $\log_3 27 - \log_3 9$ as $\frac{\log_3 27}{\log_3 9}$. The solution should be

$\log_3 27 - \log_3 9 = \log_3\left(\frac{27}{9}\right)$

$= \log_3 3$

$= 1$

15. $\log_5 5^3 + \left(\frac{2}{3}\right)\log_5 125 = \log_5 5^3 + \log_5 5^2$

$= 3 + 2$

$= 5$

16. $4\log_2 4 + \log_2 16^2 = 4(2) + 2\log_2 16$

$= 8 + 2(4)$

$= 16$

17. e.g., Because exponents and logarithms are two ways to express the same meaning, the laws are related. They follow the same pattern.

The product law of logarithms is related to the product law for exponents.

$\log_a mn = \log_a m + \log_a n \rightarrow a^{xy} = a^x a^y$; multiply two powers, add the exponents

$\log (100 \cdot 10) = \log 100 + \log 10$, which corresponds to $10^2 \cdot 10^1 = 10^{2+1}$

The quotient law of logarithms is related to the quotient law for exponents.

$\log_a \frac{m}{n} = \log_a m - \log_a n \rightarrow \frac{a^x}{a^y} = a^{x-y}$; divide two powers, subtract the exponents

$\log \frac{100}{10} = \log 100 - \log 10$, which corresponds to

$\frac{10^2}{10^1} = 10^{2-1}$

The power law of logarithms is related to the power law for exponents.

$\log_a m^n = n \log_a m \rightarrow (a^x)^y = a^{xy}$; power of a power, multiply the exponents $\log_2 16^3 = 3\log_2 16$, which corresponds to $(2^4)^3 = 2^{4 \cdot 3}$

18. e.g., agree:

$\log (xy)^2 = 2\log(xy)$

$= 2(\log x + \log y)$

$= 2\log x + 2\log y$

19. $\log(x^2 - 4x + 4) - \log(x^2 - 4)$

$= \log\left(\frac{x^2 - 4x + 4}{x^2 - 4}\right)$

$= \log\left(\frac{(x-2)(x-2)}{(x-2)(x+2)}\right)$

$= \log\left(\frac{x-2}{x+2}\right)$

$= \log(x-2) - \log(x+2)$

Or $\log(x^2 - 4x + 4) - \log(x^2 - 4)$

$= \log(x-2)^2 - \log((x-2)(x+2))$

$= 2\log(x-2) - \log(x-2) - \log(x+2)$

$= \log(x-2) - \log(x+2)$

20. $\log_x x^{m-1} = (m-1)\log_x x$

$= m\log_x x - 1\log_x x$

$= m - 1$

Math in Action, page 448

i) $\log 2 = 0.301...$, which is a little less than $\frac{1}{3}$. This result explains the position of log 2 on the scale, because log 10 = 1 and log 2 is about one-third of log 10.

ii) $\log 3 \doteq 0.477...$, which is a little less than $\frac{1}{2}$. This result explains the position of log 3 on the scale, because log 3 is about one-half of log 10.

- The scale starts at 1 because log 1 = 0.
- log 0 is undefined, since no number raised to an exponent yields zero.
- $\log 1.5 = 0.176...$, which is slightly less than $\frac{1}{5}$. This means that log 1.5 should be placed about $\frac{1}{5}$ of the way between 1 and 10.

x	$\log x$	Distance from 1 on a 13 cm Scale (cm)
1	0.00	0.0
2	0.30	3.0
3	0.48	4.8
4	0.60	6.0
5	0.70	7.0
6	0.78	7.8
7	0.85	8.5
8	0.90	9.0
9	0.95	9.5
10	1.00	10.0
11	1.04	10.4
12	1.08	10.8
13	1.11	11.1
14	1.15	11.5
15	1.18	11.8
16	1.20	12.0
17	1.23	12.3
18	1.26	12.6
19	1.28	12.8
20	1.30	13.0

• To model division, align the dividend on the log scale at the top (top row) with the divisor on the log scale at the bottom (bottom row). The 1 of the bottom row will indicate the quotient on the top row.

Lesson 7.4: Solving Exponential Equations Using Logarithms, page 455

1. a) e.g., I estimate $x = 2.1$

$10 = 3^x$

$\log 10 = \log 3^x$

$\log 10 = x \log 3$

$\frac{\log 10}{\log 3} = x$

$2.095... = x$

$2.096 = x$

b) e.g., I estimate $x = 0.5$

$10^x = 3$

$\log 10^x = \log 3$

$x \log 10 = \log 3$

$x = \frac{\log 3}{\log 10}$

$x = 0.477...$

$x = 0.477$

c) e.g., I estimate $x = 2.3$

$27 = 4^x$

$\log 27 = \log 4^x$

$\log 27 = x \log 4$

$\frac{\log 27}{\log 4} = x$

$2.377... = x$

$2.377 = x$

d) e.g., I estimate $x = 2.6$

$78 = 5^x$

$\log 78 = \log 5^x$

$\log 78 = x \log 5$

$\frac{\log 78}{\log 5} = x$

$2.706... = x$

$2.707 = x$

e) e.g., I estimate $x = 2.2$

$4.5^x = 30$

$\log 4.5^x = \log 30$

$x \log 4.5 = \log 30$

$x = \frac{\log 30}{\log 4.5}$

$x = 2.261...$

$x = 2.261$

f) e.g., I estimate $x = -0.9$

$e^x = 0.4$

$\ln e^x = \ln 0.4$

$x \ln e = \ln 0.4$

$x = \ln 0.4$

$x = -0.916...$

$x = -0.916...$

2. a) e.g., I estimate $x = 1.7$

$35 = 5(3^x)$

$7 = 3^x$

$\log 7 = \log 3^x$

$\log 7 = x \log 3$

$\frac{\log 7}{\log 3} = x$

$1.771... = x$

$1.771 = x$

b) e.g., I estimate $x = 5.3$

$117 = 3(2^x)$

$39 = 2^x$

$\log 39 = \log 2^x$

$\log 39 = x \log 2$

$\frac{\log 39}{\log 2} = x$

$5.285... = x$

$5.285 = x$

c) e.g., I estimate $x = 9.1$

$1000 = 500(1 + 0.08)^x$

$2 = 1.08^x$

$\log 2 = \log 1.08^x$

$\log 2 = x \log 1.08$

$\frac{\log 2}{\log 1.08} = x$

$9.006... = x$

$9.006 = x$

d) e.g., I estimate $x = 1.2$

$$45 = 100\left(\frac{1}{2}\right)^x$$
$$0.45 = 0.5^x$$
$$\log 0.45 = \log 0.5^x$$
$$\log 0.45 = x \log 0.5$$
$$\frac{\log 0.45}{\log 0.5} = x$$
$$1.152... = x$$
$$1.152 = x$$

3. a) e.g., between 2 and 3

$$\log_3 12 = \frac{\log 12}{\log 3}$$
$$= 2.261...$$
$$= 2.262$$

b) e.g., between 0 and 1

$$\log_8 3 = \frac{\log 3}{\log 8}$$
$$= 0.528...$$
$$= 0.528$$

c) e.g., between 3 and 4

$$\log_{\left(\frac{1}{2}\right)} 0.1 = \frac{\log 0.1}{\log\left(\frac{1}{2}\right)}$$
$$= 3.321...$$
$$= 3.322$$

d) e.g., between 1 and 2

$$\log_9 75 = \frac{\log 75}{\log 9}$$
$$= 1.964...$$
$$= 1.965$$

4. $A = P(1 + i)^n$

$$3500 = 2500(1 + 0.06)^n$$
$$1.4 = 1.06^n$$
$$\log 1.4 = \log 1.06^n$$
$$\frac{\log 1.4}{\log 1.06} = n$$
$$5.774... = n$$

It will take about 6 years for Jonah's balance to surpass $3500.

5. a) $\log_5 50 = \frac{\log 50}{\log 5}$

$$= 2.430...$$
$$= 2.431$$

b) $\log_2\left(\frac{5}{8}\right) = \frac{\log\left(\frac{5}{8}\right)}{\log 2}$

$$= -0.678...$$
$$= -0.678$$

c) $\log_3 1000 = \frac{\log 1000}{\log 3}$

$$= 6.287...$$
$$= 6.288$$

d) $\log_{0.1} 200 = \frac{\log 200}{\log 0.1}$

$$= -2.301...$$
$$= -2.301$$

6. a) $5^{x+1} = 24$

$$\log 5^{x+1} = \log 24$$
$$(x + 1) \log 5 = \log 24$$
$$x + 1 = \frac{\log 24}{\log 5}$$
$$x + 1 = 1.974...$$
$$x = 0.974...$$
$$x = 0.97$$

b) $3 = 8^{2x-1}$

$$\log 3 = \log 8^{2x-1}$$
$$\log 3 = (2x - 1) \log 8$$
$$2x - 1 = \frac{\log 3}{\log 8}$$
$$2x - 1 = 0.528...$$
$$x = 0.764...$$
$$x = 0.76$$

c) $\left(\frac{1}{3}\right)^x - 40 = 0$

$$\left(\frac{1}{3}\right)^x = 40$$
$$\log\left(\frac{1}{3}\right)^x = \log 40$$
$$x \log\left(\frac{1}{3}\right) = \log 40$$
$$x = \frac{\log 40}{\log\left(\frac{1}{3}\right)}$$
$$x = -3.357...$$
$$x = -3.36$$

d) $\left(\frac{2}{5}\right)^{-x} = 5$

$\log\left(\frac{2}{5}\right)^{-x} = \log 5$

$-x\log\left(\frac{2}{5}\right) = \log 5$

$-x = \frac{\log 5}{\log\left(\frac{2}{5}\right)}$

$-x = -1.756...$

$x = 1.76$

7. $I = 10^{(1-0.13x)}$

$4.2 = 10^{(1-0.13x)}$

$\log 4.2 = \log 10^{(1-0.13x)}$

$\log 4.2 = (1 - 0.13x)\log 10$

$\frac{\log 4.2}{\log 10} = 1 - 0.13x$

$0.623... = 1 - 0.13x$

$2.898... = x$

Plants will receive enough light at a depth of about 2.9 m.

8. e.g., Dave's error was in simplifying the right side of the equation,

$\log 5000 = \log\left(3215(1.024)^n\right)$

$\log 5000 = \log 3215 + \log 1.024^n$

$\log 5000 - \log 3215 = n\log 1.024$

$\frac{\log 5000 - \log 3215}{\log 1.024} = n$

$18.620... = n$

9. Let x represent the number of washings.
Let y represent the percentage of the original dye remaining.

$y = (1 - 0.022)^x$

$0.3 = 0.978^x$

$\log 0.3 = \log 0.978^x$

$\log 0.3 = x \log 0.978$

$\frac{\log 0.3}{\log 0.978} = x$

$54.121... = x$

Since you can't have part of a wash, 55 washes are required to give a pair of jeans the well-worn look.

10. When 75% of the wound has healed, an area of 20 cm^2 remains.

$A(t) = 80(10^{-0.023t})$

$20 = 80(10^{-0.023t})$

$0.25 = 10^{-0.023t}$

$\log 0.25 = (-0.023t)\log 10$

$\frac{\log 0.25}{\log 10} = -0.023t$

$-0.602... = -0.023t$

$26.176... = t$

Therefore 75% of the wound will be healed in about 26 days.

11. $A = P(1 + i)^n$

$35\,000 = 12\,000(1 + i)^{18}$

$2.916... = (1 + i)^{18}$

$1.061... = 1 + i$

$0.061... = i$

Therefore the interest rate of this investment is about 6.13%.

The investment has doubled when the investment is worth \$24 000.

$A = P(1 + i)^n$

$24\,000 = 12\,000(1 + 0.061...)^n$

$2 = (1.061...)^n$

$\log 2 = \log (1.061...)^n$

$\log 2 = n \log(1.061...)$

$\frac{\log 2}{\log 1.061...} = n$

$11.655... = n$

Therefore his investment doubled in about 12 years.

12. The investment has tripled when the investment is worth \$3000.

$A = P(1 + i)^n$

$3000 = 1000(1 + \frac{0.06}{12})^n$

$3 = (1.005)^n$

$\log 3 = \log (1.005)^n$

$\log 3 = n \log(1.005)$

$\frac{\log 3}{\log 1.005} = n$

$220.271... = n$

Therefore the investment tripled in about 221 months or 18 years 5 months.

13. His debt will have doubled when the balance is \$4000.

$A = P(1 + i)^n$

$4000 = 2000(1 + \frac{0.185}{12})^n$

$2 = (1.015...)^n$

$\log 2 = \log (1.015...)^n$

$\log 2 = n \log(1.015...)$

$\frac{\log 2}{\log 1.015...} = n$

$45.306... = n$

Therefore his debt will have doubled in about 4 years.

14. $A_0 = 50$, $A = 35$, $t = 3$

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$35 = 50\left(\frac{1}{2}\right)^{\frac{3}{h}}$$

$$0.7 = \left(\frac{1}{2}\right)^{\frac{3}{h}}$$

$$\log 0.7 = \log\left(\frac{1}{2}\right)^{\frac{3}{h}}$$

$$\log 0.7 = \left(\frac{3}{h}\right)\log\left(\frac{1}{2}\right)$$

$$\frac{\log 0.7}{\log\left(\frac{1}{2}\right)} = \frac{3}{h}$$

$$0.514... = \frac{3}{h}$$

$$(0.514...)h = 3$$

$$h = \frac{3}{0.514...}$$

$$h = 5.830...$$

Therefore the half-life of BPA in blood is about 5.8 h.

15. a)
$$\begin{aligned} 2^{x+1} &= 3^{x-1} \\ \log 2^{x+1} &= \log 3^{x-1} \\ (x+1)\log 2 &= (x-1)\log 3 \\ x\log 2 + \log 2 &= x\log 3 - \log 3 \\ x\log 2 - x\log 3 &= -\log 2 - \log 3 \\ x(\log 2 - \log 3) &= -\log 2 - \log 3 \\ x &= \frac{-\log 2 - \log 3}{\log 2 - \log 3} \\ x &= 4.419... \\ x &= 4.42 \end{aligned}$$

b)
$$\begin{aligned} 6^{x-6} &= 3^{x+1} \\ \log 6^{x-6} &= \log 3^{x+1} \\ (x-6)\log 6 &= (x+1)\log 3 \\ x\log 6 - 6\log 6 &= x\log 3 + \log 3 \\ x\log 6 - x\log 3 &= 6\log 6 + \log 3 \\ x(\log 6 - \log 3) &= 6\log 6 + \log 3 \\ x &= \frac{6\log 6 + \log 3}{\log 6 - \log 3} \\ x &= 17.094... \\ x &= 17.09 \end{aligned}$$

c)
$$\begin{aligned} 10^{x+1} &= 5^{x-1} \\ \log 10^{x+1} &= \log 5^{x-1} \\ (x+1)\log 10 &= (x-1)\log 5 \\ x\log 10 + \log 10 &= x\log 5 - \log 5 \\ x+1 &= x\log 5 - \log 5 \\ x - x\log 5 &= -1 - \log 5 \\ x(1 - \log 5) &= -1 - \log 5 \\ x &= \frac{-1 - \log 5}{1 - \log 5} \\ x &= -5.643... \\ x &= -5.64 \end{aligned}$$

d)
$$\begin{aligned} 2^{2x-1} &= 5^{x+1} \\ \log 2^{2x-1} &= \log 5^{x+1} \\ (2x-1)\log 2 &= (x+1)\log 5 \\ 2x\log 2 - \log 2 &= x\log 5 + \log 5 \\ 2x\log 2 - x\log 5 &= \log 2 + \log 5 \\ x(2\log 2 - \log 5) &= \log 2 + \log 5 \\ x &= \frac{\log 2 + \log 5}{2\log 2 - \log 5} \\ x &= -10.318... \\ x &= -10.32 \end{aligned}$$

16. a)
$$\begin{aligned} 3^{x+1} &= 5^{x-1} \\ \log 3^{x+1} &= \log 5^{x-1} \\ (x+1)\log 3 &= (x-1)\log 5 \\ x\log 3 + \log 3 &= x\log 5 - \log 5 \\ x\log 3 - x\log 5 &= -\log 3 - \log 5 \\ x(\log 3 - \log 5) &= -\log 3 - \log 5 \\ x &= \frac{-\log 3 - \log 5}{\log 3 - \log 5} \\ x &= 5.301... \\ x &= 5.30 \end{aligned}$$

b)
$$\begin{aligned} 2^{x+3} &= 3^{2x-1} \\ \log 2^{x+3} &= \log 3^{2x-1} \\ (x+3)\log 2 &= (2x-1)\log 3 \\ x\log 2 + 3\log 2 &= 2x\log 3 - \log 3 \\ x\log 2 - 2x\log 3 &= -3\log 2 - \log 3 \\ x(\log 2 - 2\log 3) &= -3\log 2 - \log 3 \\ x &= \frac{-3\log 2 - \log 3}{\log 2 - 2\log 3} \\ x &= 2.112... \\ x &= 2.11 \end{aligned}$$

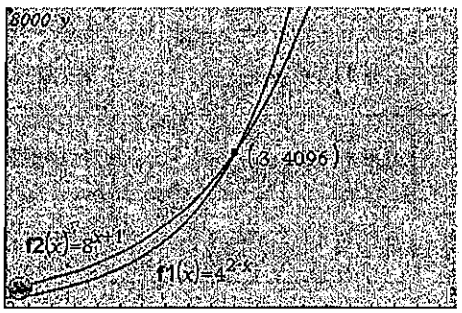
c)
$$\begin{aligned} 2^{x+1} - 3^{x-1} &= 0 \\ 2^{x+1} &= 3^{x-1} \\ \log 2^{x+1} &= \log 3^{x-1} \\ (x+1)\log 2 &= (x-1)\log 3 \\ x\log 2 + \log 2 &= x\log 3 - \log 3 \\ x\log 2 - x\log 3 &= -\log 2 - \log 3 \\ x(\log 2 - \log 3) &= -\log 2 - \log 3 \\ x &= \frac{-\log 2 - \log 3}{\log 2 - \log 3} \\ x &= 4.419... \\ x &= 4.42 \end{aligned}$$

d)
$$\begin{aligned} 6^{x-2} &= 3^{x+1} \\ \log 6^{x-2} &= \log 3^{x+1} \\ (x-2)\log 6 &= (x+1)\log 3 \\ x\log 6 - 2\log 6 &= x\log 3 + \log 3 \\ x\log 6 - x\log 3 &= 2\log 6 + \log 3 \\ x(\log 6 - \log 3) &= 2\log 6 + \log 3 \\ x &= \frac{2\log 6 + \log 3}{\log 6 - \log 3} \\ x &= 6.754... \\ x &= 6.75 \end{aligned}$$

17. Strategy 1: Write both sides of the equation as powers with equivalent bases, then solve for the unknown by comparing exponents.

$$4^{2x} = 8^{x+1}$$
$$2^{2(2x)} = 2^{3(x+1)}$$
$$2^{4x} = 2^{3x+3}$$
$$4x = 3x + 3$$
$$x = 3$$

Strategy 2: Use a system of equations and graphing technology to determine the intersection of the two curves.



Therefore, $x = 3$

Strategy 3: Take the common logarithm of both sides of the equation, and then use the logarithm laws to solve for the unknown.

$$4^{2x} = 8^{x+1}$$
$$\log 4^{2x} = \log 8^{x+1}$$
$$(2x)\log 4 = (x+1)\log 8$$
$$\frac{x+1}{2x} = \frac{\log 4}{\log 8}$$
$$\frac{1}{2} + \frac{1}{2x} = 0.666...$$
$$\frac{1}{2x} = 0.666... - \frac{1}{2}$$
$$x = \frac{1}{2(0.166...)}$$
$$x = 3$$

18. a)
$$\begin{aligned} 3^x - 3^{x-1} &= 20 \\ 3^{x-1}(3-1) &= 20 \\ 3^{x-1}(2) &= 20 \\ 3^{x-1} &= 10 \\ \log 3^{x-1} &= \log 10 \\ (x-1)\log 3 &= 1 \\ x - 1 &= \frac{1}{\log 3} \\ x - 1 &= 2.095... \\ x &= 3.10 \end{aligned}$$

b)
$$\begin{aligned} 5^{x+2} + 5^x &= 104 \\ 5^x(5^2 + 1) &= 104 \\ 5^x(26) &= 104 \\ 5^x &= 4 \\ \log 5^x &= \log 4 \\ x \log 5 &= \log 4 \\ x &= \frac{\log 4}{\log 5} \\ x &= 0.861... \\ x &= 0.86 \end{aligned}$$

19.
$$2^{10^x} = e^6$$
$$\ln 2^{10^x} = \ln e^6$$
$$10^x \ln 2 = 6 \ln e$$
$$10^x \ln 2 = 6$$
$$10^x = \frac{6}{\ln 2}$$
$$\log 10^x = \log 8.656...$$
$$x \log 10 = \log 8.656...$$
$$x = \frac{\log 8.656...}{\log 10}$$
$$x = 0.937...$$
$$x = 0.937$$

20. a)
$$\begin{aligned} 3^{x+2} &= 5(5^{x-1}) \\ 3^{x+2} &= 5^x \\ \log 3^{x+2} &= \log 5^x \\ (x+2)\log 3 &= x \log 5 \\ x \log 3 + 2 \log 3 &= x \log 5 \\ x \log 3 - x \log 5 &= -2 \log 3 \\ x(\log 3 - \log 5) &= -2 \log 3 \\ x &= \frac{-2\log 3}{\log 3 - \log 5} \\ x &= 4.301... \\ x &= 4.30 \end{aligned}$$

b)
$$\begin{aligned} 2(2^{x+3}) &= 3(3^{2x-1}) \\ 2^{x+4} &= 3^{2x} \\ \log 2^{x+4} &= \log 3^{2x} \\ (x+4)\log 2 &= 2x \log 3 \\ x \log 2 + 4 \log 2 &= 2x \log 3 \\ x \log 2 - 2x \log 3 &= -4 \log 2 \\ x(\log 2 - 2 \log 3) &= -4 \log 2 \\ x &= \frac{-4\log 2}{\log 2 - 2\log 3} \\ x &= 1.843... \\ x &= 1.84 \end{aligned}$$

History Connection, page 459

A. $A = P(1 + i)^n$
$A = 1(1 + 1)^1$
$A = 2$

B. If the growth is compounded semiannually, then $i = \frac{1}{2}$. So, the compound interest formula becomes

$$A = 1\left(1 + \frac{1}{2}\right)^2$$

$A = 2.25$
$A = 2.25$

C. I set up a table to record the number of compounding periods per year and the year-end balances.

Number of Compounding Periods per Year	Compound Interest Formula	A
3	$A = 1\left(1 + \frac{1}{3}\right)^3$	2.370...
4	$A = 1\left(1 + \frac{1}{4}\right)^4$	2.441...
5	$A = 1\left(1 + \frac{1}{5}\right)^5$	2.488...
⋮	⋮	⋮
10	$A = 1\left(1 + \frac{1}{10}\right)^{10}$	2.593...
⋮	⋮	⋮
20	$A = 1\left(1 + \frac{1}{20}\right)^{20}$	2.653...
⋮	⋮	⋮
50	$A = 1\left(1 + \frac{1}{50}\right)^{50}$	2.691...
⋮	⋮	⋮
100	$A = 1\left(1 + \frac{1}{100}\right)^{100}$	2.704...
⋮	⋮	⋮
1000	$A = 1\left(1 + \frac{1}{1000}\right)^{1000}$	2.716...
⋮	⋮	⋮
10 000	$A = 1\left(1 + \frac{1}{10\,000}\right)^{10\,000}$	2.718...

I noticed that as the number of compounding periods increases, the balance approaches e.

D. From my table of values, I knew that when $n = 10\,000$, $A = 2.718\,145...$. So, I used trial-and-error to determine a value for n where $A = 2.718\,28$.

Number of Compounding Periods per Year	Compound Interest Formula	A
20 000	$A = 1\left(1 + \frac{1}{20\,000}\right)^{20\,000}$	2.718 213...
40 000	$A = 1\left(1 + \frac{1}{40\,000}\right)^{40\,000}$	2.718 247...
60 000	$A = 1\left(1 + \frac{1}{60\,000}\right)^{60\,000}$	2.718 259...
200 000	$A = 1\left(1 + \frac{1}{200\,000}\right)^{200\,000}$	2.718 275...
400 000	$A = 1\left(1 + \frac{1}{400\,000}\right)^{400\,000}$	2.718 278...
800 000	$A = 1\left(1 + \frac{1}{800\,000}\right)^{800\,000}$	2.718 280...

It would take about 800 000 compounding periods per year to get an amount equal to e accurate to 5 decimal places.

E. I knew the values of P_0 and k.
$P_0 = 6.91$ billion
$k = 0.0114$
$t = 2020 - 2011$ or 9
I substituted the given values into the continuous growth equation.
$P = P_0 \cdot e^{kt}$
$P = 6.91e^{(0.0114)(9)}$
$P = 7.656...$
The projected world population would be about 7.66 billion in 2020, according to this model.

F. I solved the continuous growth equation for t.

$$P = P_0 \cdot e^{kt}$$

$$\frac{P}{P_0} = e^{kt}$$

$$\ln\left(\frac{P}{P_0}\right) = \ln e^{kt}$$

$\ln e^{kt}$ means "the exponent of e that gives e^{kt}." I used this fact to simplify the right side.

$$\ln\left(\frac{P}{P_0}\right) = kt$$

$$\left(\frac{1}{k}\right)\ln\left(\frac{P}{P_0}\right) = t$$

To determine the time it takes for the world population to double, I knew that $\frac{P}{P_0} = \frac{2P_0}{P_0}$, which is 2. I

substituted this value and $k = 0.0114$ into the equation.

$$\left(\frac{1}{0.0114}\right)\ln 2 = t$$

$$t = 60.802...$$

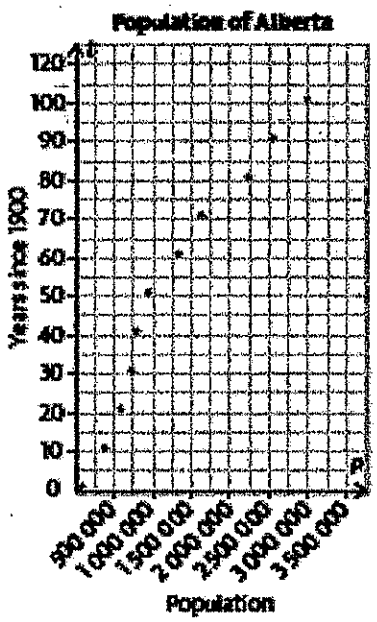
According to this model, the world population will be double the population in 2011 after about 61 years, or in 2072.

Lesson 7.5: Modelling Data Using Logarithmic Functions, page 466

1. a) e.g., Graph is logarithmic because graph is increasing, x-intercept is 1, there is no y-intercept, end behaviour is QIV to QI, domain is $\{x \mid x > 0, x \in R\}$, and the range is $\{y \mid y \in R\}$.
b) When the decibel level increases by 10, the relative sound intensity increases by a factor of 10.
c) When the relative sound intensity doubles, the decibel level increases by a factor of 3.

2. The logarithmic regression equation that models this data is $y = -6.653... + 108.491...\ln x$.
x-intercept: 1.063...
y-intercept: none
End behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
Function: increasing

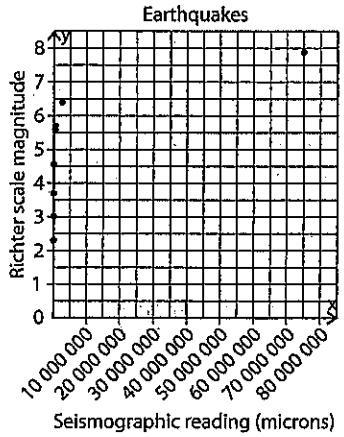
3. a)



b) The regression equation that models the data is $t = -346.090... + 28.957...\ln P$
P-intercept: 155 076.923...
t-intercept: none
End behaviour: QIV to QI
Domain: $\{P \mid P > 0, P \in W\}$
Range: $\{t \mid t \geq 0, t \in W\}$
Function: increasing

c) $t = -346.090... + 28.957...\ln P$
$t = -346.090... + 28.957...\ln(2\,000\,000)$
$t = 74.043...$
The population exceeded 2 000 000 in 1974.

4. a) The independent variable is seismographic reading and the dependent variable is Richter Scale magnitude.
b)



c) The regression equation for the data is $M = -0.006... + 0.434...\ln r$.
d) $M = -0.006... + 0.434...\ln r$

Let $M = 5.7$
$5.7 = -0.006... + 0.434...\ln r_1$
$5.706... = 0.434...\ln r_1$
$13.122... = \ln r_1$
$r_1 = e^{13.122...}$

Let $M = 4.5$
$4.5 = -0.006... + 0.434...\ln r_2$
$4.506... = 0.4348\ln r_2$
$10.362... = \ln r_2$
$r_2 = e^{10.362...}$

$$\frac{r_1}{r_2} = \frac{e^{13.122...}}{e^{10.362...}}$$

$$\frac{r_1}{r_2} = e^{(13.122...-10.362...)}$$

$$\frac{r_1}{r_2} = e^{2.759...}$$

$$\frac{r_1}{r_2} = 15.794...$$

A magnitude 5.7 earthquake is about 15.8 times more intense than a magnitude 4.5 earthquake.

5. a) The independent variable is pressure and the dependent variable is altitude.
b) The regression equation for this data is $h = 30\,665.960... - 6640.436...\ln P$.

c) P-intercept: 101.297...
h-intercept: none
End behaviour: QI to QIV
Domain: $\{P \mid P > 0, P \in R\}$
Range: $\{h \mid h \in R\}$
Function: decreasing

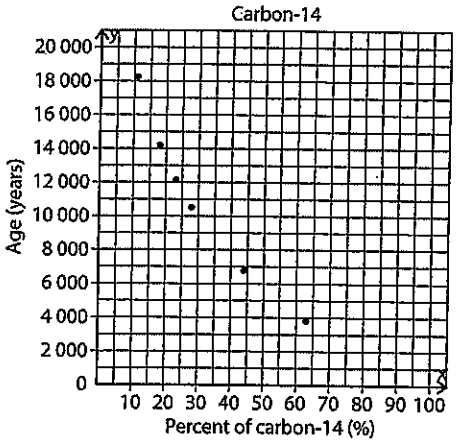
d)
$$y = 30\,665.960... - 6640.436... \ln x$$
$$139 = 30\,665.960... - 6640.436... \ln x$$
$$-30\,526.960... = -6640.436... \ln x$$
$$4.597... = \ln x$$
$$x = e^{4.597...}$$
$$x = 99.199...$$

The pressure setting that Michael will need to use is 99.2 kPa.

e)
$$y = 30\,665.960... - 6640.436... \ln x$$
$$8848 = 30\,665.960... - 6640.436... \ln x$$
$$-21\,817.960... = -6640.436... \ln x$$
$$3.285... = \ln x$$
$$x = e^{3.285...}$$
$$x = 26.725...$$

The atmospheric pressure at the summit of Mt. Everest is 26.7 kPa.

6. a) Here is the graph.



The regression equation that represents this data is $t = 38\,070.332... - 8266.961... \ln P$

b) $t = 38\,070.332... - 8266.961... \ln P$
$t = 38\,070.332... - 8266.961... \ln (96.8)$
$t = 268.437...$
The age of these fragments is about 268 years.

c) $t = 38\,070.332... - 8266.961... \ln P$
$t = 38\,070.332... - 8266.961... \ln (50)$
$t = 5729.790...$
The half-life of carbon-14 is 5730 years.

7. a) The exponential regression equation that represents this data is $A = 14\,999.826...(1.045...)^t$.

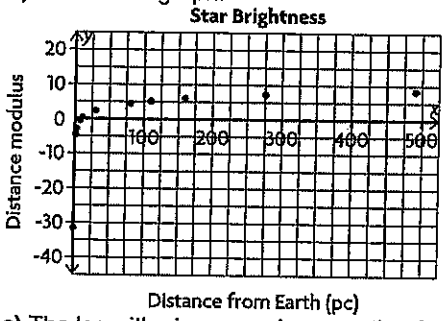
b) The logarithmic regression equation that represents this data is $t = -218.442... + 22.717... \ln A$.

c) Exponential:
$$A = 14\,999.826...(1.045...)^t$$
$$25\,000 = 14\,999.826...(1.045...)^t$$
$$1.666... = (1.045...)^t$$
$$\log 1.666... = \log (1.045...)^t$$
$$\log 1.666... = t \log 1.045...$$
$$\frac{\log 1.666...}{\log 1.045...} = t$$
$$11.604... = t$$

Logarithmic:
$t = -218.442... + 22.717... \ln A$
$t = -218.442... + 22.717... \ln (25\,000)$
$t = 11.604...$
It will take about 12 years for the balance to equal $25 000 according to both equations.
e.g., I prefer using the logarithmic because it makes the calculations simpler.

8. a) The independent variable is distance from Earth and the dependent variable is distance modulus.

b) Here is the graph.



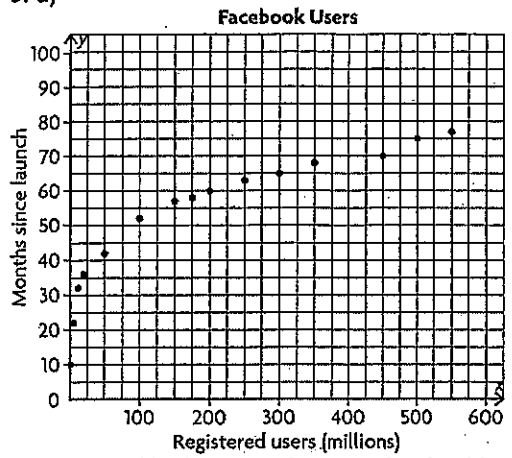
c) The logarithmic regression equation for this data is $m = -5.086... + 2.174... \ln d$.

d) $m = -5.086... + 2.174... \ln d$
$m = -5.086... + 2.174... \ln (2.39)$
$m = -3.191...$
The distance modulus of Wolf 359 is –3.19.

e)
$$m = -5.086... + 2.174... \ln d$$
$$-1.66 = -5.086... + 2.174... \ln d$$
$$3.426... = 2.174... \ln d$$
$$1.575... = \ln d$$
$$e^{1.575...} = d$$
$$d = 4.834...$$

Gliese 876 is 4.8 pc from Earth.

9. a)



b) The logarithmic regression equation for this data is $t = 5.888... + 10.833... \ln n$.

c) $t = 5.888... + 10.833... \ln n$

$t = 5.888... + 10.833... \ln (275)$

$t = 66.736...$

Facebook first surpassed 275 million registered users during the 66th month after February 2004, which is August 2009.

10. Enter the data in my calculator, perform a logarithmic regression, graph the data points in a scatter plot and the logarithmic function on the same axes, and identify the point with the known x-value and the unknown y-value.

11. a) i)

$$A = P(1+i)^n$$

$$A = 5000(1+0.05)^n$$

$$\frac{A}{5000} = 1.05^n$$

$$\ln\left(\frac{A}{5000}\right) = \ln 1.05^n$$

$$\ln A - \ln 5000 = n \ln 1.05$$

$$n = \frac{\ln A - \ln 5000}{\ln 1.05}$$

ii)

$$A = P(1+i)^n$$

$$A = 5000\left(1+\frac{0.05}{2}\right)^{2n}$$

$$\frac{A}{5000} = 1.025^{2n}$$

$$\ln\left(\frac{A}{5000}\right) = \ln 1.025^{2n}$$

$$\ln A - \ln 5000 = 2n \ln 1.025$$

$$2n = \frac{\ln A - \ln 5000}{\ln 1.025}$$

$$n = \frac{\ln A - \ln 5000}{2 \ln 1.025}$$

iii)

$$A = P(1+i)^n$$

$$A = 5000\left(1+\frac{0.05}{4}\right)^{4n}$$

$$\frac{A}{5000} = 1.0125^{4n}$$

$$\ln\left(\frac{A}{5000}\right) = \ln 1.0125^{4n}$$

$$\ln A - \ln 5000 = 4n \ln 1.0125$$

$$n = \frac{\ln A - \ln 5000}{4 \ln 1.0125}$$

iv)

$$A = P(1+i)^n$$

$$A = 5000\left(1+\frac{0.05}{365}\right)^{365n}$$

$$\frac{A}{5000} = 1.000\,136...^{365n}$$

$$\ln\left(\frac{A}{5000}\right) = \ln 1.000\,136...^{365n}$$

$$\ln A - \ln 5000 = 365n \ln 1.000\,136...$$

$$n = \frac{\ln A - \ln 5000}{365 \ln 1.000\,136...}$$

b) e.g., The denominator approaches $\ln 1 = 0$ as the number of compounding periods per year increases.

Applying Problem-Solving Strategies, page 472

D. It looks as if you need to ask no more than 10 questions to guess a number from 1 to 1000.

E. Let n represent the numbers remaining and let q represent the number of questions asked.
Since the secret number is from 1 to 1000, it means that there are 1000 possible numbers.
Each time you do a binary search, you are dividing the set of possible numbers in half.
I wrote a table of values for the first few guesses.

n	q
1000	0
$1000\left(\frac{1}{2}\right)$	1
$\left[1000\left(\frac{1}{2}\right)\right]\left(\frac{1}{2}\right) = 1000\left(\frac{1}{2}\right)^2$	2
$\left[1000\left(\frac{1}{2}\right)^2\right]\left(\frac{1}{2}\right) = 1000\left(\frac{1}{2}\right)^3$	3

I noticed that the exponent on the base $\left(\frac{1}{2}\right)$ is equal to q, the number of questions asked. So, I set up an equation expressing n in terms of q to model the situation.

$$n = 1000\left(\frac{1}{2}\right)^q$$

F. I substituted $n = 1$ into my equation from prompt E.

$$1 = 1000\left(\frac{1}{2}\right)^q$$

$$\frac{1}{1000} = \left(\frac{1}{2}\right)^q$$

$$q = \log_{\left(\frac{1}{2}\right)}\left(\frac{1}{1000}\right)$$

$$q = 9.965...$$

You would need to ask 10 questions to find a number from 1 to 1000. This answer matches my answer in prompt D.

G. To determine the number of questions needed to guess a number from 1 to one billion, I changed 1000 to 1 000 000 000 in my equation.

$$n = 1\,000\,000\,000\left(\frac{1}{2}\right)^q$$

Then I substituted $n = 1$ and solved for q:

$$1 = 1\,000\,000\,000\left(\frac{1}{2}\right)^q$$

$$\frac{1}{1\,000\,000\,000} = \left(\frac{1}{2}\right)^q$$

$$q = \log_{\left(\frac{1}{2}\right)}\left(\frac{1}{1\,000\,000\,000}\right)$$

$$q = 29.897...$$

You would need to ask 30 questions to guess a number from 1 to 1 000 000 000.

H. Answers will vary. e.g., One possibility is to allow the guesser to divide the list of remaining numbers into three groups, instead of two. Questions would now be like this: "Is your number between a and b, b and c, or c and d?" This would make the game go much more quickly, since each turn reduces the number of remaining numbers by $\frac{2}{3}$.

To guess a number from 1 to 1000, the equation that would model the situation would now be

$$n = 1000\left(\frac{1}{3}\right)^q$$

$$1 = 1000\left(\frac{1}{3}\right)^q$$

$$\frac{1}{1000} = \left(\frac{1}{3}\right)^q$$

$$q = \log_{\left(\frac{1}{3}\right)}\left(\frac{1}{1000}\right)$$

$$q = 6.287...$$

Chapter Self-Test, page 473

1. x-intercept: 1
y-intercept: none
End behaviour: QI to QIV
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
Function: decreasing

2. i) b, e.g., x-intercept is 1, no y-intercept, QIV to QI. Thus, the function is logarithmic and increasing so the correct choice is b.
ii) c, e.g., x-intercept is 1, no y-intercept, QI to QIV. Thus, the function is logarithmic and decreasing so the correct choice is c.
iii) a, e.g., no x-intercept, y-intercept is 2, decreasing. Thus, the function is exponential so the correct choice is a.

3. a) $x = 5^y$
$y = \log_5 x$
b) $y = \log_6 x$
$x = 6^y$

4. a) $\log_2 2^5 = 5 \log_2 2$
$= 5$
b) $\log_{11} 121 = \log_{11} 11^2$
$= 2 \log_{11} 11$
$= 2$
c) $\log_3\left(\frac{1}{81}\right) = \log_3 3^{-4}$
$= -4 \log_3 3$
$= -4$
d) $\log_{\left(\frac{1}{2}\right)} 8 = \log_{\left(\frac{1}{2}\right)}\left(\frac{1}{2}\right)^{-3}$
$= -3\log_{\left(\frac{1}{2}\right)}\left(\frac{1}{2}\right)$
$= -3$

5. $\log_5\left(\frac{1}{25}\right) - \log_2 8 = \log_5 5^{-2} - \log_2 2^3$

$= -2 - 3$

$= -5$

6. a) $\log_2 320 - \log_2 10 = \log_2\left(\frac{320}{10}\right)$

$= \log_2 32$

$= 5$

b) $\log_3 10 - \log_3 8.1 = \log_3 (10 \cdot 8.1)$

$= \log_3 81$

$= 4$

c) $2\log_4 8 = \log_4 8^2$

$= \log_4 64$

$= 3$

7. a) $3 \log 3 + 3 \log 2 = 3 \log (3 \cdot 2)$

$= 3 \log 6$

$= 2.334...$

$= 2.33$

b) $4 \log 5 - \log 1 = 4 \log 5$

$= 2.795...$

$= 2.80$

8. a) $3^{x-1} = 215$

$\log 3^{x-1} = \log 215$

$(x-1)\log 3 = \log 215$

$x - 1 = \frac{\log 215}{\log 3}$

$x - 1 = 4.888...$

$x = 5.888...$

$x = 5.889$

b) $5^{x+2} = 7^{x-1}$

$\log 5^{x+2} = \log 7^{x-1}$

$(x+2)\log 5 = (x-1)\log 7$

$x \log 5 + 2 \log 5 = x \log 7 - \log 7$

$x \log 5 - x \log 7 = -2 \log 5 - \log 7$

$x(\log 5 - \log 7) = -2 \log 5 - \log 7$

$x = \frac{-2\log 5 - \log 7}{\log 5 - \log 7}$

$x = 15.349...$

$x = 15.350$

9. $\log_3 36 = \frac{\log 36}{\log 3}$

$= 3.261...$

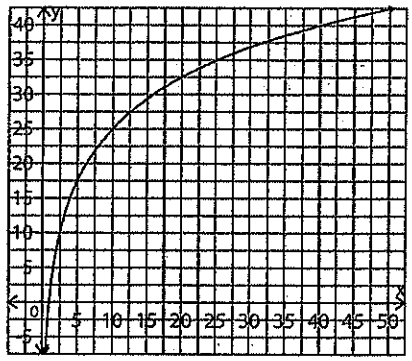
$= 3.26$

10. Let x represent the energy released. Let y represent the magnitude of the earthquake.
The regression equation is
$y = -1.199... + 0.289...\ln x$.
$y = -1.199... + 0.289...\ln x$
$y = -1.199... + 0.289...\ln (1.1 \cdot 10^{16})$
$y = 9.493...$
The Valdivia earthquake had a magnitude of about 9.5.

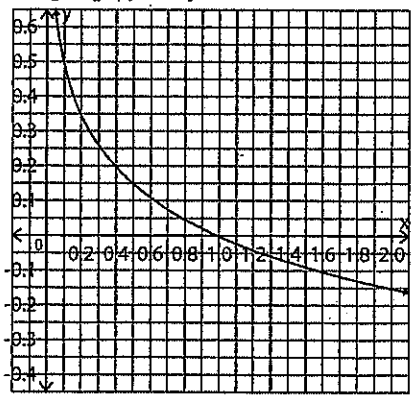
Chapter Review, page 476

1. x-intercept: 1
Number of y-intercepts: none
End behaviour: QI to QIV
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
Function: decreasing

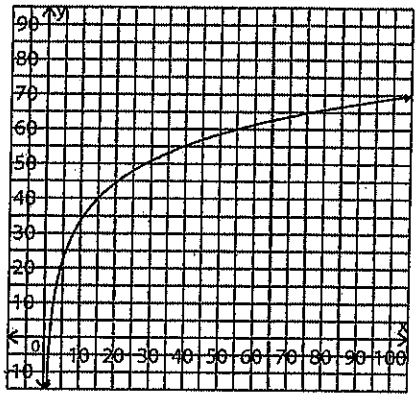
2. a) x-intercept: 1
Number of y-intercepts: none
End behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$



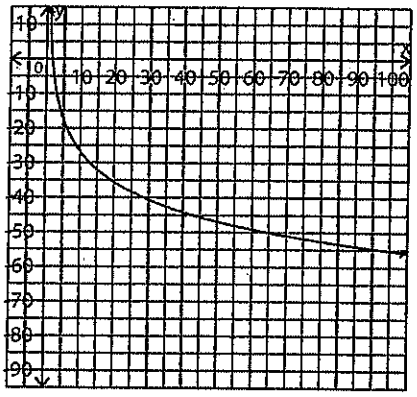
b) x-intercept: 1
Number of y-intercepts: none
End behaviour: QI to QIV
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$



c) x-intercept: 1
Number of y-intercepts: none
End behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$



d) x-intercept: 1
Number of y-intercepts: none
End behaviour: QI to QIV
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$



3. a) $\log 200 = 2.301\ldots$
$= 2.30$
b) $\ln 5 = 1.609\ldots$
$= 1.61$

4. a) $\log 200 = 2.30$ because $10^{2.30} = 200$
b) $\ln 5 = 1.61$ because $e^{1.61} = 5$

5. a) $x = 4^y$
$y = \log_4 x$
b) $y = \log_7 x$
$x = 7^y$

6. a) $\log_8 64 = \log_8 8^2$
$= 2$
b) $\log_6 1 = \log_6 6^0$
$= 0$
c) $\log\left(\frac{1}{100}\right) = \log 10^{-2}$
$= -2$

d) $\log_{\left(\frac{1}{4}\right)} 16 = \log_{\left(\frac{1}{4}\right)}\left(\frac{1}{4}\right)^{-2}$
$= -2$

7. $2\log_5 25 - 4\log_5 125 = 2\log_5 5^2 - 4\log_5 5^3$
$= (2)(2) - (4)(3)$
$= 4 - 12$
$= -8$

8. a) $\log_6 108 - \log_6 3 = \log_6\left(\frac{108}{3}\right)$
$= \log_6 36$
$= 2$
b) $\log 4 + 2\log 5 = \log 4 + \log 5^2$
$= \log (4 \cdot 25)$
$= \log 100$
$= 2$

9. a) e.g.,

	Exponents	Logarithms
Product Law	$b^m b^n = b^{m+n}$	$\log_b mn = \log_b m + \log_b n$
Quotient Law	$\frac{b^m}{b^n} = b^{m-n}$	$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$
Power Law	$\left(b^m\right)^n = b^{m \cdot n}$	$\log_b m^n = n\log_b m$

b) e.g., Product Law: $4^3 \cdot 4^5 = 4^{3+5}$,
$\log_6 (8)(4) = \log_6 8 + \log_6 4$
Quotient Law: $\frac{4^5}{4^3} = 4^{5-3}$, $\log_6\left(\frac{8}{5}\right) = \log_6 8 - \log_6 5$
Power Law: $(4^3)^4 = 4^{3 \cdot 4}$, $\log_6 4^3 = 3\log_6 4$

10. a)
$6^x = 95$
$\log 6^x = \log 95$
$x \log 6 = \log 95$
$x = \frac{\log 95}{\log 6}$
$x = 2.541\ldots$
$x = 2.542$

b)
$25(1.5)^x = 100$
$1.5^x = 4$
$\log 1.5^x = \log 4$
$x \log 1.5 = \log 4$
$x = \frac{\log 4}{\log 1.5}$
$x = 3.419\ldots$
$x = 3.419$

c)
$2^{2x+2} = 500$
$\log 2^{2x+2} = \log 500$
$(2x + 2)\log 2 = \log 500$
$2x + 2 = \frac{\log 500}{\log 2}$
$2x + 2 = 8.965\ldots$
$2x = 6.965\ldots$
$x = 3.482\ldots$
$x = 3.483$

11.

$$\begin{aligned} 3^x &= 5^{x+2} \\ \log 3^x &= \log 5^{x+2} \\ x \log 3 &= (x+2)\log 5 \\ x \log 3 &= x \log 5 + 2 \log 5 \\ x \log 3 - x \log 5 &= 2 \log 5 \\ x(\log 3 - \log 5) &= 2 \log 5 \\ x &= \frac{2\log 5}{\log 3 - \log 5} \\ x &= -6.301\ldots \\ x &= -6.301 \end{aligned}$$

12.

$$\begin{aligned} A &= P(1+i)^n \\ 7500 &= 5500(1+0.045)^n \\ 1.363\ldots &= (1.045)^n \\ \log 1.363\ldots &= \log (1.045)^n \\ \log 1.363\ldots &= n \log 1.045 \\ n &= \frac{\log 1.363\ldots}{\log 1.045} \\ n &= 7.046\ldots \end{aligned}$$

It will take about 7 years for the balance to surpass $7500.

13. a) $\log_5 60 = \dfrac{\log 60}{\log 5}$

$= 2.543\ldots$

$= 2.54$

b) $\log_8 146 = \dfrac{\log 146}{\log 8}$

$= 2.396\ldots$

$= 2.40$

14. The logarithmic regression equation of this data is $y = 151.211\ldots - 32.836\ldots (\ln x)$.

$$\begin{aligned} y &= 151.211\ldots - 32.836\ldots(\ln x) \\ 200 &= 151.211\ldots - 32.836\ldots(\ln x) \\ 48.788\ldots &= -32.836\ldots(\ln x) \\ -1.485\ldots &= \ln x \\ x &= e^{-1.485\ldots} \\ x &= 0.226\ldots \end{aligned}$$

Therefore about 0.23% of the sunlight penetrates water to a depth of 200 m.

Chapter Task, page 477

B. e.g.,

Leading Digit (d)	$1+\frac{1}{d}$	Frequency of d	Probability of d
1	$1+\frac{1}{1}=2$	9	$\frac{9}{77}=0.116\ldots$ or 11.688...%
2	$1+\frac{1}{2}=1.5$	3	$\frac{3}{77}=0.038\ldots$ or 3.896...%
3	$1+\frac{1}{3}=1.333\ldots$	17	$\frac{17}{77}=0.220\ldots$ or 22.077...%
4	$1+\frac{1}{4}=1.25$	13	$\frac{13}{77}=0.168\ldots$ or 16.883...%
5	$1+\frac{1}{5}=1.2$	11	$\frac{11}{77}=0.142\ldots$ or 14.285...%
6	$1+\frac{1}{6}=1.166\ldots$	10	$\frac{10}{77}=0.129\ldots$ or 12.987...%
7	$1+\frac{1}{7}=1.142\ldots$	6	$\frac{6}{77}=0.077\ldots$ or 7.792...%
8	$1+\frac{1}{8}=1.125$	3	$\frac{3}{77}=0.038\ldots$ or 3.896...%
9	$1+\frac{1}{9}=1.111\ldots$	5	$\frac{5}{77}=0.064\ldots$ or 6.493...%

C. If the first two data points are removed from the data, then the equation of the logarithmic regression function that models the probabilities of the remaining leading digits, as a function of $\left(1+\frac{1}{d}\right)$, is

$$P = -0.041\ldots + (0.945\ldots)\ln\left(1+\frac{1}{d}\right)$$

I think excluding these two digits is reasonable, because of the nature of the data.

D. I calculated the probability of each digit from the regression equation.

Leading Digit (d)	$1+\frac{1}{d}$	Value of P from Regression Equation
1	$1+\frac{1}{1}=2$	$-0.041... + (0.945...)\ln 2$ $= 0.613...$ or $61.317...\%$
2	$1+\frac{1}{2}=1.5$	$-0.041... + (0.945...)\ln 1.5$ $= 0.341...$ or $34.126...\%$
3	$1+\frac{1}{3}=1.333...$	$-0.041... + (0.945...)\ln 1.333...$ $= 0.229...$ or $22.993...\%$
4	$1+\frac{1}{4}=1.25$	$-0.041... + (0.945...)\ln 1.25$ $= 0.168...$ or $16.893...\%$
5	$1+\frac{1}{5}=1.2$	$-0.041... + (0.945...)\ln 1.2$ $= 0.130...$ or $13.035...\%$
6	$1+\frac{1}{6}=1.166...$	$-0.041... + (0.945...)\ln 1.166...$ $= 0.103...$ or $10.372...\%$
7	$1+\frac{1}{7}=1.142...$	$-0.041... + (0.945...)\ln 1.142...$ $= 0.084...$ or $8.423...\%$
8	$1+\frac{1}{8}=1.125$	$-0.041... + (0.945...)\ln 1.125$ $= 0.069...$ or $6.935...\%$
9	$1+\frac{1}{9}=1.111...$	$-0.041... + (0.945...)\ln 1.111...$ $= 0.057...$ or $5.761...\%$

E. I compared my calculated results of P with those of Benford's law.

Value of P from Benford's Law (%)	My Value of P from Regression Equation (%)
30.1	61.317...
17.6	34.126...
12.5	22.993...
9.7	16.893...
7.9	13.035...
6.7	10.372...
5.8	8.423...
5.1	6.935...
4.6	5.761...

Without counting my first two data points, I noticed that my values of P are higher than those in Benford's law. The reason for this is that there were not enough data points and it appears that several points were neither on, nor very close to, the regression curve.

Chapter 7 Diagnostic Test, TR page 437

1. a) $2^x = 16$
$2^x = 2^4$
$x = 4$

b) $10^x = 0.000\,01$
$10^x = 10^{-5}$
$x = -5$

c) $5^x = 125$
$5^x = 5^3$
$x = 3$

d) $25^x = 1$
$25^x = 25^0$
$x = 0$

2. a) Since $(2^4 = 16) \le (2^x = 30) \le (2^5 = 32)$, the two integers that are closest to the value of x are 4 and 5.

b) Since $(3^3 = 27) \le (3^x = 75) \le (3^4 = 81)$, the two integers that are closest to the value of x are 3 and 4.

c) Since $(12^2 = 144) \le (12^x = 280) \le (12^3 = 1728)$, the two integers that are closest to the value of x are 2 and 3.

d) Since $(10^3 = 1000) \le (10^x = 9425) \le (10^4 = 10\,000)$, the two integers that are closest to the value of x are 3 and 4.

3. a)
$$4^x = 28$$
$$\log 4^x = \log 28$$
$$x \log 4 = \log 28$$
$$x = \frac{\log 28}{\log 4}$$
$$x = 2.403...$$
$$x = 2.40$$

b)
$$1.5^x = 15.63$$
$$\log 1.5^x = \log 15.63$$
$$x \log 1.5 = \log 15.63$$
$$x = \frac{\log 15.63}{\log 1.5}$$
$$x = 6.780...$$
$$x = 6.78$$

c)
$$\left(\frac{1}{2}\right)^x = 10$$
$$\log\left(\frac{1}{2}\right)^x = \log 10$$
$$x\log\left(\frac{1}{2}\right) = 1$$
$$x = \frac{1}{\log\left(\frac{1}{2}\right)}$$
$$x = -3.321...$$
$$x = -3.32$$

d)
$$0.65^x = 0.0019$$
$$\log 0.65^x = \log 0.0019$$
$$x \log 0.65 = \log 0.0019$$
$$x = \frac{\log 0.0019}{\log 0.65}$$
$$x = 14.545...$$
$$x = 14.55$$

4. a) Let y represent the intensity of light, as a percent. Let x represent the depth, in metres. Exponential functions of the form $y = a(b)^x$. The initial amount of sunlight at the surface is 75 so $a = 75$.

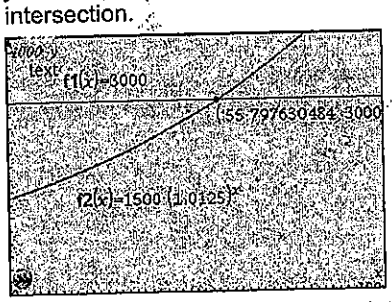
The intensity of the remaining light decreases by 5% for every metre of depth so
$b = 1 - 0.05$
$= 0.95$
Therefore the equation that models this relationship is $y = 75(0.95)^x$, where y represents the intensity of light, as a percent, at a depth x, in metres.

b) $y = 75(0.95)^x$
$y = 75(0.95)^5$
$y = 58.033...$
Therefore about 58% of sunlight penetrates to a depth of 5 m.

c) $y = 75(0.95)^x$
$10 = 75(0.95)^x$
$0.133... = 0.95^x$
$\log 0.133... = \log 0.95^x$
$\log 0.133... = x \log 0.95$
$$x = \frac{\log 0.133...}{\log 0.95}$$
$x = 39.281...$
Therefore the sunlight will be only 10% of its original intensity at a depth of about 39 m.

5. a) $A = P(1+i)^n$
$$A = 1500\left(1+\frac{0.025}{2}\right)^{20}$$
$A = 1923.055...$
Therefore $1923.06 will be in the account after 10 years.

b) To determine when the deposit will grow to $3000, we need to solve the equation $3000 = 1500(1.0125)^n$ for n. To do this, we graph $y = 3000$ and $y = 1500(1.0125)^x$ to determine the point of intersection.



Therefore the graphs intersect at (55.797..., 3000). It will take 55.797... compounding periods or about 28 years for the deposit to be worth $3000.

6. $P(t) = 125(0.945)^n$
$85 = 125(0.945)^n$
$0.68 = 0.945^n$
$\log 0.68 = \log 0.945^n$
$\log 0.68 = n \log 0.945$
$$n = \frac{\log 0.68}{\log 0.945}$$
$n = 6.817...$
The population will decrease to 85 in the year 2017.

Review of Terms and Connections, TR page 439

1. a) vi)
b) ii)
c) i)
d) iii)
e) vii)
f) iv)
g) v)

2. a)

x	y	Ratio
1	6	
2	18	3
3	54	3
4	162	3
5	486	3

Therefore the ratio between consecutive y-values is 3. Since the ratio is greater than 1, the function models exponential growth.

b)

x	y	Ratio
1	200	
2	100	$\frac{1}{2}$
3	50	$\frac{1}{2}$
4	25	$\frac{1}{2}$
5	12.5	$\frac{1}{2}$

Therefore the ratio between consecutive y-values is $\frac{1}{2}$. Since the ratio is less than 1, the function models exponential decay.

3. a) Using a graphing calculator, the equation for the exponential regression is $y = 2(3)^x$.
b) Using a graphing calculator, the equation for the exponential regression is $y = 400\left(\frac{1}{2}\right)^x$.

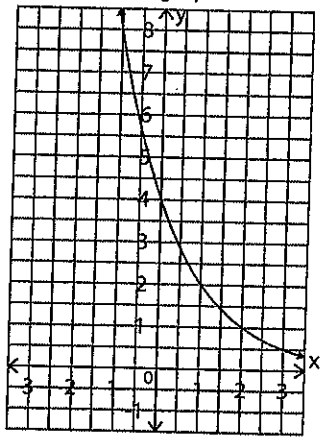
4. a) Since $b > 1$, the function models exponential growth. The rate of growth is b, which is 4.
b) Since $b > 1$, the function models exponential growth. The rate of growth is b, which is 6.
c) Since $b < 1$, the function models exponential decay. The rate of decay is b, which is 0.4.
d) Since $b > 1$, the function models exponential growth. The rate of growth is b, which is 2.
e) Since $b < 1$, the function models exponential decay. The rate of decay is b, which is $\frac{1}{4}$.
f) Since $b > 1$, the function models exponential growth. The rate of growth is b, which is 3.

5. a) Number of x-intercepts: none
y-intercept: 5

End behaviour: curve extends from quadrant II to quadrant I
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y > 0, y \in R\}$
Increasing or decreasing: increasing
b) Number of x-intercepts: none
y-intercept: 1
End behaviour: curve extends from quadrant II to quadrant I
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y > 0, y \in R\}$
Increasing or decreasing: increasing
c) Number of x-intercepts: none
y-intercept: 1
End behaviour: curve extends from quadrant II to quadrant I
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y > 0, y \in R\}$
Increasing or decreasing: decreasing
d) Number of x-intercepts: none
y-intercept: $\frac{1}{2}$
End behaviour: curve extends from quadrant II to quadrant I
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y > 0, y \in R\}$
Increasing or decreasing: increasing
e) Number of x-intercepts: none
y-intercept: 2
End behaviour: curve extends from quadrant II to quadrant I
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y > 0, y \in R\}$
Increasing or decreasing: decreasing
f) Number of x-intercepts: none
y-intercept: $\frac{1}{7}$
End behaviour: curve extends from quadrant II to quadrant I
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y > 0, y \in R\}$
Increasing or decreasing: increasing

6. a) Number of x-intercepts: none
y-intercept: 2
End behaviour: curve extends from quadrant II to quadrant I
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y > 0, y \in R\}$
Increasing or decreasing: increasing
Rate of growth: 5
b) Number of x-intercepts: none
y-intercept: 1500
End behaviour: curve extends from quadrant II to quadrant I
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y > 0, y \in R\}$
Increasing or decreasing: increasing
Rate of growth: 1.05
c) Number of x-intercepts: none
y-intercept: 50
End behaviour: curve extends from quadrant II to quadrant I
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y > 0, y \in R\}$
Increasing or decreasing: decreasing
Rate of decay: 0.97

7. Here is the graph.



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1. x-intercept: 1
Number of y-intercepts: none
End behaviour: the curve extends from quadrant IV to quadrant I
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$

2. a) $128 = 15^x$
$x = \log_{15} 128$
b) $\frac{1}{16} = 8^x$
$x = \log_8 \frac{1}{16}$
c) $0.45 = 5^x$
$x = \log_5 0.45$
d) $4 = 9^x$
$x = \log_9 4$

3. a) e.g., I estimate $x = 1.5$

$x = \log_{15} 128$

$x = \dfrac{\log 128}{\log 15}$

$x = 1.791...$

$x = 1.79$

b) e.g., I estimate $x = -1.5$

$x = \log_8 \left(\dfrac{1}{16}\right)$

$x = \dfrac{\log\left(\dfrac{1}{16}\right)}{\log 8}$

$x = -1.333...$

$x = -1.33$

c) e.g., I estimate $x = -0.5$

$x = \log_5 0.45$

$x = \dfrac{\log 0.45}{\log 5}$

$x = -0.496...$

$x = -0.50$

d) e.g., I estimate $x = 0.6$

$x = \log_9 4$

$x = \dfrac{\log 4}{\log 9}$

$x = 0.630...$

$x = 0.63$

4. a) Function a) is increasing, because $a > 1$. The only graph that shows an increasing logarithmic function is ii). So, function a) matches graph ii).

b) The graph for function iii) extends from quadrant I to quadrant IV and has an x-intercept but no y-intercepts. These are characteristics of a logarithmic function. Function b) matches graph iii).

c) The graph for function i) extends from quadrant II to quadrant I and has a y-intercept but no x-intercepts. These are characteristics of an exponential function. Function c) matches graph i).

5. $\log_4 64 - \log 100 = \log_4 4^3 - \log 10^2$

$= 3 - 2$

$= 1$

6. a) $\log_5 375 - \log_5 3 = \log_5 \left(\dfrac{375}{3}\right)$

$= \log_5 125$

$= \log_5 5^3$

$= 3$

b) $\log_2 19.2 + \log_2 5 = \log_2 (19.2 \cdot 5)$

$= \log_2 96$

$= \dfrac{\log 96}{\log 2}$

$= 6.584...$

$= 6.585$

7. a) $4 \log 2 - 2 \log 5 = \log 2^4 - \log 5^2$

$= \log 16 - \log 25$

$= \log \dfrac{16}{25}$

$= -0.193...$

$= -0.194$

b) $-4 \log 5 + 2 \log 3 = \log 5^{-4} + \log 3^2$

$= \log (5^{-4})(3^2)$

$= \log \dfrac{9}{625}$

$= -1.841...$

$= -1.842$

8. a) $5^{x+3} = 450$

$\log 5^{x+3} = \log 450$

$(x + 3) \log 5 = \log 450$

$x + 3 = \dfrac{\log 450}{\log 5}$

$x = \dfrac{\log 450}{\log 5} - 3$

$x = 0.795...$

$x = 0.796$

b) $4^{x+3} = 6^{x-1}$

$\log 4^{x+3} = \log 6^{x-1}$

$(x + 3) \log 4 = (x - 1) \log 6$

$x \log 4 + 3 \log 4 = x \log 6 - \log 6$

$\log 6 + 3 \log 4 = x \log 6 - x \log 4$

$\log 6 + \log 4^3 = x(\log 6 - \log 4)$

$\log 6(64) = x \log \left(\dfrac{6}{4}\right)$

$\log 384 = x \log 1.5$

$x = \dfrac{\log 384}{\log 1.5}$

$x = 14.676...$

$x = 14.676$

9. The equation of the logarithmic regression is $y = -1.199... + 0.289...(\ln x)$ where x is the energy released, in kJ, and y is the magnitude of the earthquake.

$y = -1.199... + 0.289...(\ln x)$

$9.0 = -1.199... + 0.289...(\ln x)$

$10.199... = 0.289...(\ln x)$

$35.232... = \ln x$

$x = e^{35.232...}$

$x = 2.000... \times 10^{15}$

About 2.0×10^{15} kJ of energy was released during the earthquake that occurred off the coast of Japan in March 2011.